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The Influence of Spring Design upon Spring Rate

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THE INFLUENCE OF SPRING DESIGN UPON SPRING RATE

by

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LIFE

Velton Curtis White, Jr. was born in Syracuse, New York on June 14, 1933. He graduated from Elyria High School, Elyria, Ohio, 1950.

He received a Bachelor of Science degree from Baldwin Wallace College, Beria, Ohio in 1957. He attended Loyola University School of Dentistry and received a Doctor of Dental Surgery Degree in June, 1961.

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CHAPTER I

Introduction

A. Introductory Remarks and Statements of the Problem:

Orthodontics is a bio-mechanical science. Biologically a pressure of low magnitude and constant duration over a wide range is generally accepted as desirable. Mechanically this can be achieved by means of a spring appliance which delivers a continuous but slowly decreasing force magnitude with a wide range of deflection.

The force versus deflection values for different, high-modulus wire appliances are not accurately known.

It is the purpose of this study to analyze the force versus deflection values of various designs of wire springs in order to learn how the design factors influence spring rates.

B. Review of Literature:

Angle (1907) stated the best results in moving a tooth are obtained when the degree of force is only that necessary to bring about physiological changes in the tissues. The proper physiological force in the appliance is when the force is no greater than a pressure which gives a "snug feeling". A force greater than a "snug feeling" is defeating the purpose for it induces pathological instead of physiological changes in the tissues which cause inflammation and pain.

Oppenheim (1911) showed that there was physiological tooth movement and pathological tooth movement. He did not, however, state the magnitude of the force which would give these movements.

Fish (1917) pointed out the fact that treatment of a malocclusion by mechanical means can be conducted rationally only by due observance of engineering principles. He also presented a few of the basic laws of mechanics applicable to appliances for moving teeth. He states that malocclusion is a biological problem and that clinical experience in study of tissue behavior during and after treatment is important, but the orthodontists cannot afford to construct and operate mechanical appliances with their customary disregard of the laws of physics. Fish, in the paper, presents the idea that analytical mechanics is a pure science consisting in mathematical expression and interpretation of the natural laws governing force and motion which can be divided up as: "kinematics which is motion without reference to the forces; dynamics which is the mathematical treatment of forces and other circumstances of motion; dynamics is branched into kinetics which is variable motion and, statics which deals with uniform motion and rest". Fish also presents technical mechanics as the application of these laws in the practice of engineering. In conclusion, Fish makes the following statement. "Until orthodontists outgrow the use of indeterminate appliances, and borrow from engineering the practice of laying out on paper what you propose to do before you try to do it orthodontia will continue to be purely experimental".

Irish (1927) states mechanics must always play a great role in accomplishing every orthodontic procedure. Pressure is the means of treatment in the field of orthodontics and should be constructive in its application.

McKeag (1929) states that when a wire is deflected by pressure exerted by a force, the wire will emit the same pressure in its attempt to return to its original position. He found by taking a round piece of wire 18 cm. in length and .35 mm. thick fixed at one end and free elsewhere that if it is deviated 5 mm. by pressure it will exert a force of approximately 20 gm. before it can return to its position of rest. The wire does not travel 5 mm. all along its length in order to return to the rest position and, therefore, does not exert a uniform force of 20 gm. at each point. In practice he found a pressure of approximately 2 ounces was a convenient one for application to a single tooth. He stated that the lower pressure brought about a slower tooth movement but raising the pressure beyond a certain point would not bring about any increase of speed of movement.

Brumfield (1930) in doing work on structural features related to orthodontic materials and appliances indicates there is a lack of investigation and knowledge of forces which are to come on the structures in orthodontics and the effects of those forces in inducing stresses in the various members of the structure. He used laws of elasticity of materials and mathematics to comprise tables of: load, deflections, torques, of various wires and springs. He noted that the flexibility varies directly with the load and the square of the length and inversely with the modulus of elasticity and the cross-sectional dimension. Since the square of the length of the beam is involved, the length then becomes the most important variables all affect flexibility according to the first power of the variable and equal percentages of change in those variables would have approximately equal percentages of effect on the flexibility. It can be concluded that two structures designed for identical purposes would be subject to the same loads, the one structure of high-strength material would be as flexible or more flexible only by reducing the cross-sectional dimensions.

Schwartz (1932) examined histologically the effect of various forces on the surrounding tissues of three premolar teeth of a dog. He applied known forces to the teeth in a buccal direction by means of a calibrated springs attached to a lingual arch. After movement of the teeth had occurred, he examined the surrounding tissue histiologically for the effect of the various forces. He concluded from this that a constant gentle pressure of 20 - 26 gms/sq.cm. can bring about tooth movement damage to tissues.

Moore and Peyton (1933) measured the force and displacement of orthodontic springs. The springs were supported in a broach holder and a basket of graduated weights were then added to the spring at various measured distances from the holder. Deflections were measured by a short range telescope, with cross hairs, which was able to be raised or lowered until the cross hair was focused on the upper side of the spring each time.

Richmond (1933) measured forces with a "stress and tension" gauge which could measure up to 16 ounces in a push or pull direction. This was of clinical use but was not accurate for all experimental uses.

Oppenheim (1935 - 1936) reported that: (1) there is no purely physiological orthodontic therapy, (2) lack of pain and firmness of the teeth are the sole criteria for judging that no traumatic conditions are being effected (3) both intermittent and continuous forces create pathologic changes. In the use of the latter the damage is generally more severe and is located both on the buccal and on the lingual sides, (4) by using gentle intermittent forces, the periodontal membrane recovers comparatively quickly, this being due to a rapidly re-established blood supply. (5) inflammation is the morphologic reason for the clinical symptom of sensitiveness of orthodontically moved teeth. (6) the cementum does not need the increased resistance of osteoid bone in order to be changed by resorption. The resistance of the normal bone is sufficient to bring about resorption even in the young cementum which is considered more resistant to resorption. (7) the osteoid, formed on the side of pressure during the periods of rest, is considered by Gettlieb School as dangerous for the cementum because of its apparently greater resistance against resorption and, therefore, the intermittent form of tooth movement is discarded in favor of the continuous.

Orban (1936) stated that every orthodontic movement of a tooth by means of appliances was an overstress in the biologic sense and it was irrelevant whether an intermittent or a continuous pressure or traction was exerted. Bone and the connective tissue convering have two biologic functions: osteoclastic resorption and osteoblastic deposition. When a tooth is moved intermittently the bone newly formed during rest must be resorbed first. The root cementum is less sensitive to pressure than is bone but excessive pressure will cause root resorption. The force should also be low enough as to not interfere with the vitality of the periodontal connective tissue.

Stuteville (1937) lists the injuries which may be caused by orthodontic forces as those of: the periodontal membrane, the surfaces of the roots of teeth, the alveolar bone, the gingivae, the pulps of teeth. He states that the amount of space through which the force is active, and not the degree of orthodontic force used, is the important factor.

Wunderly and Richardson (1947) in investigating the mechanical properties of orthodontic arch wires used the following formula in experiments from which they reached the conclusion that an arch wire should possess high flexibility combined with high resilience.

The formula for the relation between load and deflection:

$$D = \frac{C\,WL^3}{EI}$$

W is the load.

L is the distance as marked.

E is the Young's modulus of elasticity.

D is the deflection of the beam at the point where the load is applied.

C is a constant the value of which depends only on the conditions of loading:

(1) One end fixed; concentrated load at other. C = 1/3.

(2) Two supports equidistant from a concentrated central load. C = 1/48.

(3) The ends of the beam are rigidly fixed and

the concentrated load is applied at the center, C = 1/192.

I. Values for Beams: Moment of Inertia:

Section of Beam	I
Circular, radius a	$1/4\ \pi a^4$
Rectangular, dimensions b x t (t in the plane of bending)	$1/12\ bt^3$
Square, dimensions t x t	$1/12\ t^4$

Some general results can be stated from the formula:

1. Wires of circular section, the deflection varies inversely as the fourth power of the diameter.
2. Wires of rectangular section, of dimensions b x t, the deflection is inversely proportional both to b and to the cube of t when t is the dimension of the beam in the plane of bending.
3. Wires of square section and dimensions of t x t the deflection in inversely proportional to the fourth power of t.

If beams are of the same material and the supports are the same distance apart in both cases so that E & L are the same for both beams, deflections of beams can be compared of different cross sections and different loads with the formula.

$$D = K.\ W/I \qquad K \text{ is a constant}$$

Storey (1952) did clinical experiments to determine the optimum force range for the movement of teeth. The experiment was the distal movement of canine teeth on five patients. Helical torsion springs were used with a heavy spring, activated to apply known loads from 400 - 600 gms. on one side of the mouth and a light spring activated to apply known loads from 175 - 300 gms. To apply these known loads which had previously been calibrated by determining load deflection curves were used, and reference marks were placed on the appliance corresponding to the deflections for various known loads. In each case the lower first molar and second bicuspid were used as anchor units. A fixed point in the upper jaw was used for reference. Canine and anchor unit measurements were taken each week by means of calipers. The change in measurements was used to determine the new value for the deflection of the springs. The spring had a known load deflection curve so the new value of load could be determined. The results were a similar behaviour of the teeth was noticed in all the five cases studied. With the light springs, movement of teeth occurred rapidly after the initial activation to the range 175 - 300 gms. This continued until the force had decreased to a value which varied from 135 - 180 gms. for the different cases. Movement then either ceased or continued at a very slow rate. In the case of the heavy springs which were activated to higher values of force, initially very little or no movement of the canine occurred. Instead the anchor unit moved in a very marked fashion until the force applied by the spring had decreased to the range 200 - 300 gms. (i.e., the canine was acting as an "anchor tooth" and the so called anchor teeth were the teeth being moved). Then movement of the anchor unit stopped and the cuspid started to move rapidly giving the same type of behaviour as found in the canine that were moved by the light springs. Storey states that the optimum range of force values which should be used to produce a maximum rate of movement of the cuspid tooth without movement of the anchor unit is from 150 to 200 gms.

Steiner (1953) in a work on power shortage and delivery in orthodontic appliances states, "A compromise must be made between extremely light appliances which possess a high degree of elasticity and store power (force) well, and heavy, non-yielding appliances which possess a high degree of stability and resist undesirable environment forces to a sufficient degree".

Johns (1953) concluded from his investigations of comparing various orthodontic mechanisms with respect to the forces delivered by them while being used to treat clinical patients, that orthodontic appliances in general and the edgewise arch in particular exert much more force than was formerly realized. He states they were pathologic in action for the majority of their forces were much higher than the normal blood pressure in the periodontal ligaments.

Reitan (1955) stated in his work on evaluation of orthodontic forces. "The appropriate amount of force to be applied may vary considerably according to the type of movement required; approximately 250 grams during the final stage of continuous bodily movement of canines, and only 25 grams in a movement of extrusion of individual front teeth should be applied".

Begg (1956) reported his method of treatment will produce universal tooth movements with a light optimum force. His appliance is a thin round wire which sets the teeth in motion simultaneously along the shortest, most direct paths to the position the teeth will occupy at completion of treatment.

Weinstein relates that the concept of equilibrium is essential to an understanding of the forces acting upon individual teeth or upon segments of the dentition. These forces include those acting upon the crowns and upon the roots.

Parker (1959) presented a historical review of appliances and pointed out they were only the means of attaching force application and would not reduce the necessity for a complete understanding of anchorage problems and cellular responses to applied force. He states that if a fraction of the biological research in orthodontics were directed toward our mechanical problems, progress could be greatly hastened.

Stoner (1960) noted that if control can be established over direction, degree, duration, and distribution of forces in orthodontic appliances then efficient tooth movement can be anticipated. Regardless of what appliance is being used, it is the application of the mechanics to move teeth without losing this control that permits the operator to obtain his results.

Burstone (1961) and co-workers, experimented with orthodontic appliances which are capable of delivering light continuous force. They considered the magnitude, direction, point of application, distance, and uniformity of force within this distance. The conclusions from the results of the experiments reported that theoretical considerations by experimental data indicated the possibility of predicting the force characteristics of an appliance by nonempirical methods.

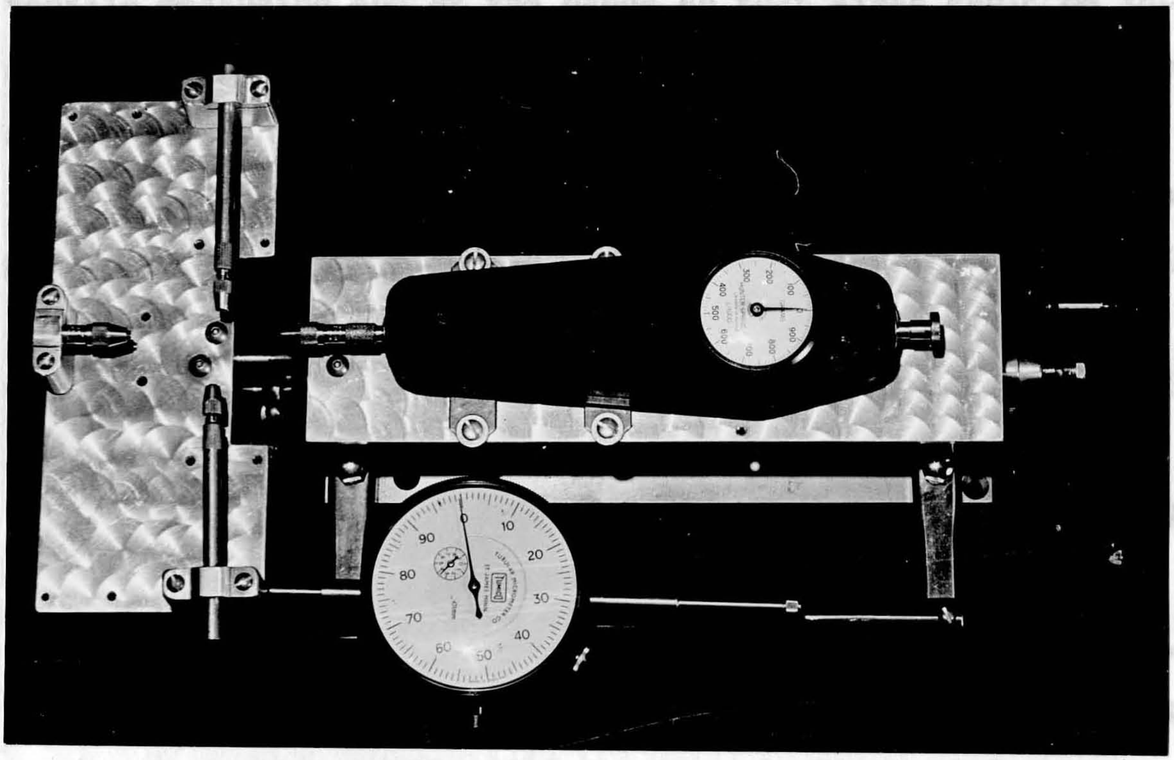
A. Mechanical Apparatus:

The instrument which was used for testing the springs was built on special order for the Orthodontic Department by the Adahold Manufacturing Company of Hammond, Indiana. A top view of this is shown in Figure 2.1 and a side elevation is shown in Figure 2.2. The instrument consisted of a heavy rectangular base upon which was mounted a fixed rectangular platform or stage (A). This stage was supported above the base on two heavy steel posts. One of these steel posts (B) was located near one end of the base while the other was near the center of the base. Extending the full length of this assembly, parallel with a longitudinal centerline, there was a carefully turned lead screw (C) having a pitch of 32 threads per inch. Riding on this lead screw there was a cross-head (D) having an adjustable nut which could be set to minimize backlash. This cross-head was attached to and was arranged to slide two parallel steel tubes (E) which were supported in the two aforementioned posts. These tubes could be extended several inches out from one end of the base assembly.

Mounted upon the extended ends of these two parallel steel tubes there was a vertical post that supported a small moveable platform or stage (F). This stage was flush with the larger fixed stage mentioned previously and was arranged to be moved to and away from the larger platform in a smoothly controlled manner by means of the lead screw and cross-head.

Extending from the opposite end of the base and platform assembly there was a crank (G) attached to the end of the lead screw, making it easy to control the turning of the screw and to obtain accurate positioning of the moveable platform with respect to the fixed platform.

There was mounted upon the fixed platform a Hunter force gauge (H) having a range of 0 - 1000 grams. The dial of this



Top View

Figure 2.1

gauge is shown in Figure 2.3. A central plunger (J) could be pushed or pulled since it extended from both ends of the gauge body. The gauge was positioned on the platform with its plunger parallel with the lead screw and the two guide tubes which carried the moveable platform. A pinvise (K) was mounted on the tensile measuring end of the gauge so that wires could be inserted axially into the collet of the vise and could be grasped securely for testing.

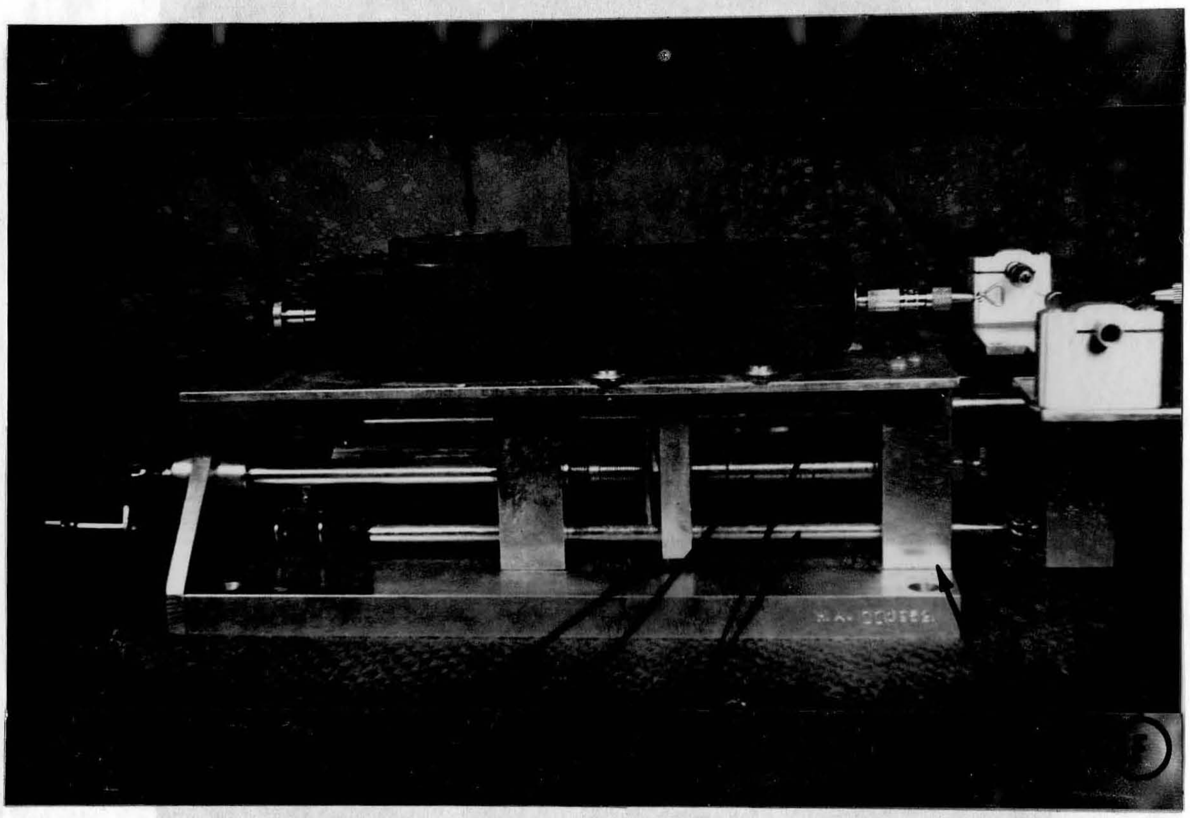
An array of holes was drilled and tapped in the moveable platform so that clamps (L) could be mounted on it in convenient locations to hold another pinvise or hooks to which springs could be attached. Figure 2.4 shows a plan view of this platform with some clamps mounted.

Translation of the moveable platform was measured by a dial indicator (Scherr-Tumico) calibrated in millimeters. This indicator is shown in Figure 2.1 but is hidden in Figure 2.2. The full range was 50 millimeters and each division on the scale represented .01 mm. The dial is shown in Figure 2.5.

B. <u>Material Used:</u>

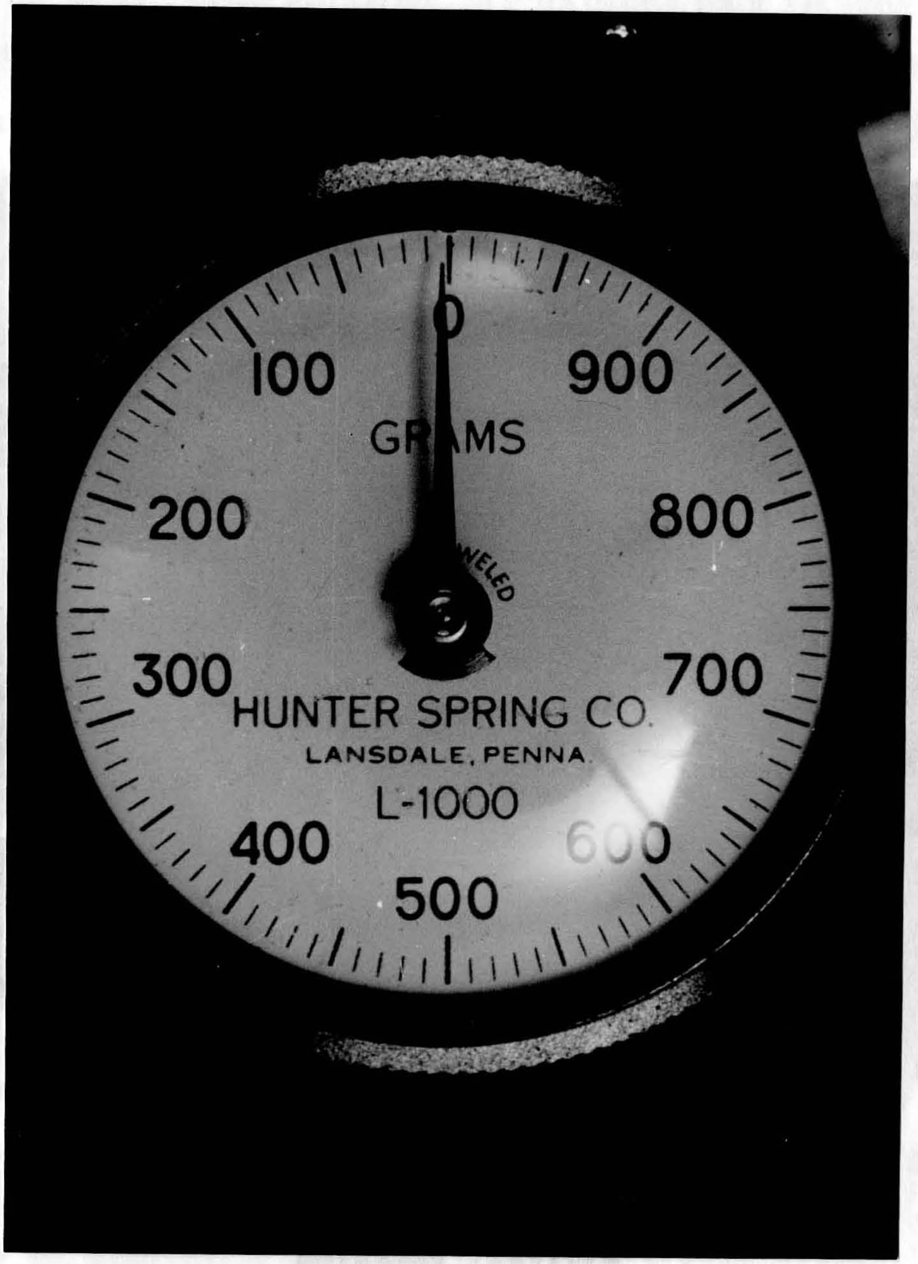
1. Composition of Elgiloy

Elgiloy is a cobalt base alloy which derives its maximum properties from a combination of cold work and heat treatment. The amount of response to heat treatment is a function of the



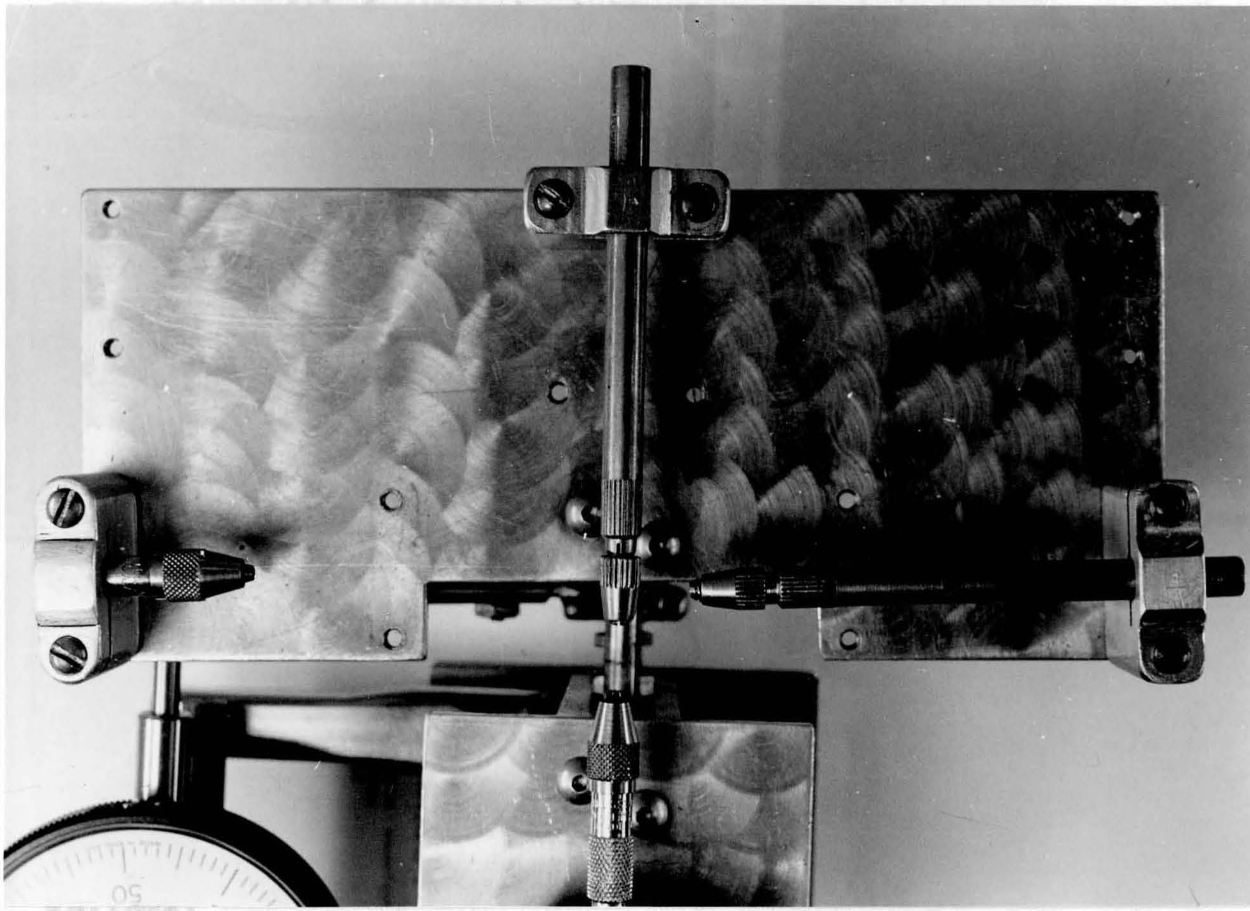
A fixed rectangular platform
B steel posts
C lead screw
D cross-head
E parallel steel tubes
F movable platform
G crank
H Hunter force gauge
J central plunger
K pinvise
L clamps

Figure 2.2



Hunter Force Gauge

Figure 2.3



Moveable Platform

Figure 2.4

amount of prior cold working. It is important to note, therefore, that once Elgiloy has been heat treated the effect of the cold work is altered and Elgiloy will not regain any greater properties with further heat treatment.

Elgiloy combines the excellent strength characteristics and corrosion resistance of cobalt with the added corrosion resistance of chromium and the strength and ductility of nickel. Molybdenym is added to increase the mechanical properties at elevated temperatures. The remaining elements provide the additional properties of hardenability and set resistance. The nominal composition of Elgiloy is as follows:

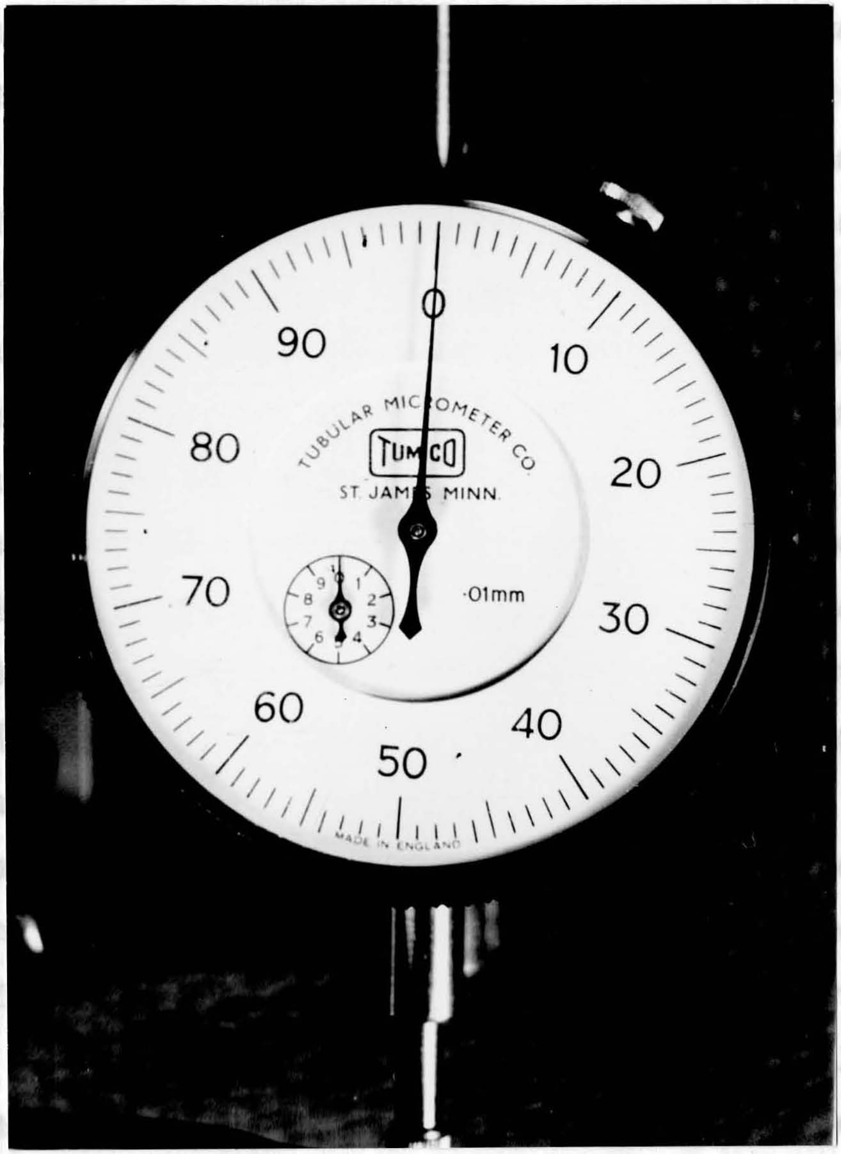
NOMINAL COMPOSITION
(Patented: U.S. Pat. No. 2524661)

Cobalt......... 40%	Manganese.........2%
Chromiumum......20%	Carbon.........0.15%
Nickel......... 15%	Beryllium......0.04%
Molybdenum...... 7%	Iron.........Balance

Mechanical properties of Elgiloy strip which has been cold reduced 85% and heat treated five hours at about 870 F. are as follows:

Proportional Limit...........................	233,000 psi.
Yield Strength...............................	280,000 psi.
Ultimate Strength............................	368,000 psi.
Modulus of Elasticity........................	29,500,000
Shear Modulus................................	12,000,000
Hardness (Vickers)...........................	702
Hardness (Rocksell C)........................	58

As mentioned above, this patented alloy has such unique



Dial Indicator
Figure 2.5

features that it merits somewhat more explanation than is available from the simple tables. Personal communication with representatives of the Elgin Watch Company, maker of the raw material and the Rocky Mountain Metal Products Company, fabricators of the wire, has revealed other pertinent information.

When a rod of metal in a dead soft condition is drawn through a series of dies it is work-hardened or cold-worked. Of course, it gets smaller in cross-sectional area and the percent reduction in area becomes a valuable measure of the amount of cold working that is done on the wire. Elgiloy has the ability to develop increased physical properties as a result of cold working and subsequent heat treating. The greater cold working (percent reduction in area) causes increasingly greater enhancement of yield strength and ultimate strength. There is a combination where the peak of improvement is reached at about 60 percent reduction in area followed by heat treatment at around 900 F. for five hours. Figure 2.6 shows that the heat treating causes a great increase in the modulus of elasticity, E, but that the reduction in area causes a gradual reduction in E.

Without quoting any specific figures, the Rocky Mountain Metal Products Company has indicated that the four strengths of wire which it sells are produced by subjecting the raw material to a known amount of cold reduction in bringing it to the stated final size. If a product were to have a final diameter of .016" and were to result from a 50 percent reduction, the diameter of the dead soft raw material must have been about .23".

This would result in a wire having a yield strength of about 220,000 psi according to the graph in Figure 2.6. The modulus of elasticity would be only 23.2 millions of psi. Optimum heat treatment could bring this up to about 28.4 millions of psi. The yeild strength could be increased to as much as 360,000 psi. In contrast to this a wire which had been produced with only 30 percent reduction in area would develop a yield strength of only 225,000 psi even after heat treating.

C. Test Specimens:

The nomenclature applicable to the physical characteristics of the test specimens is illustrated in Figure 2.7. It is necessary to define the words "arms", "legs" and "helices" as they are used in describing the parts of the various test specimens.

In preparing to fabricate the various test specimens, a known length (150 mm.), of .016" "green" Elgiloy wire was cut from the stock wire purchased from the Rocky Mountain Metal

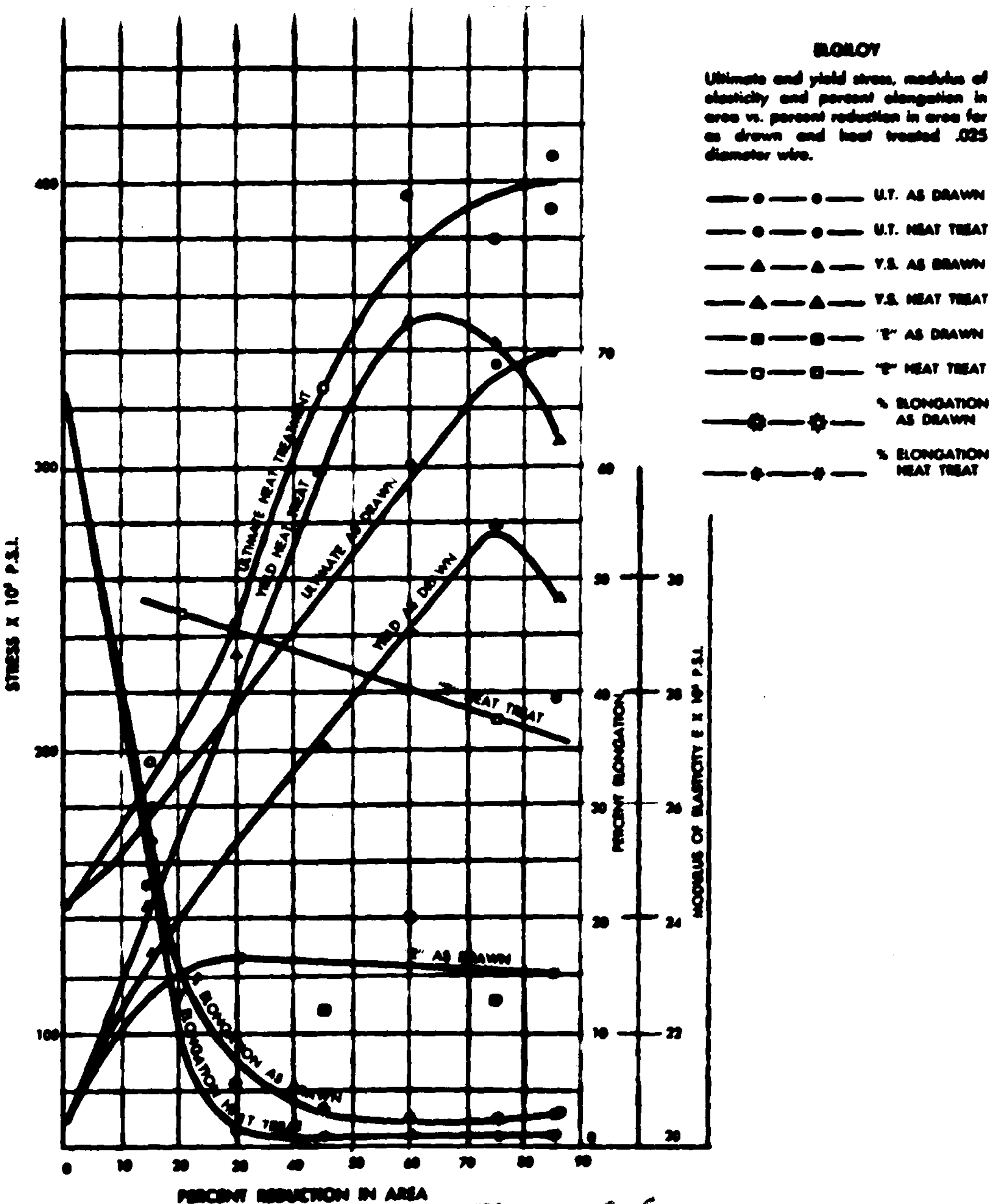
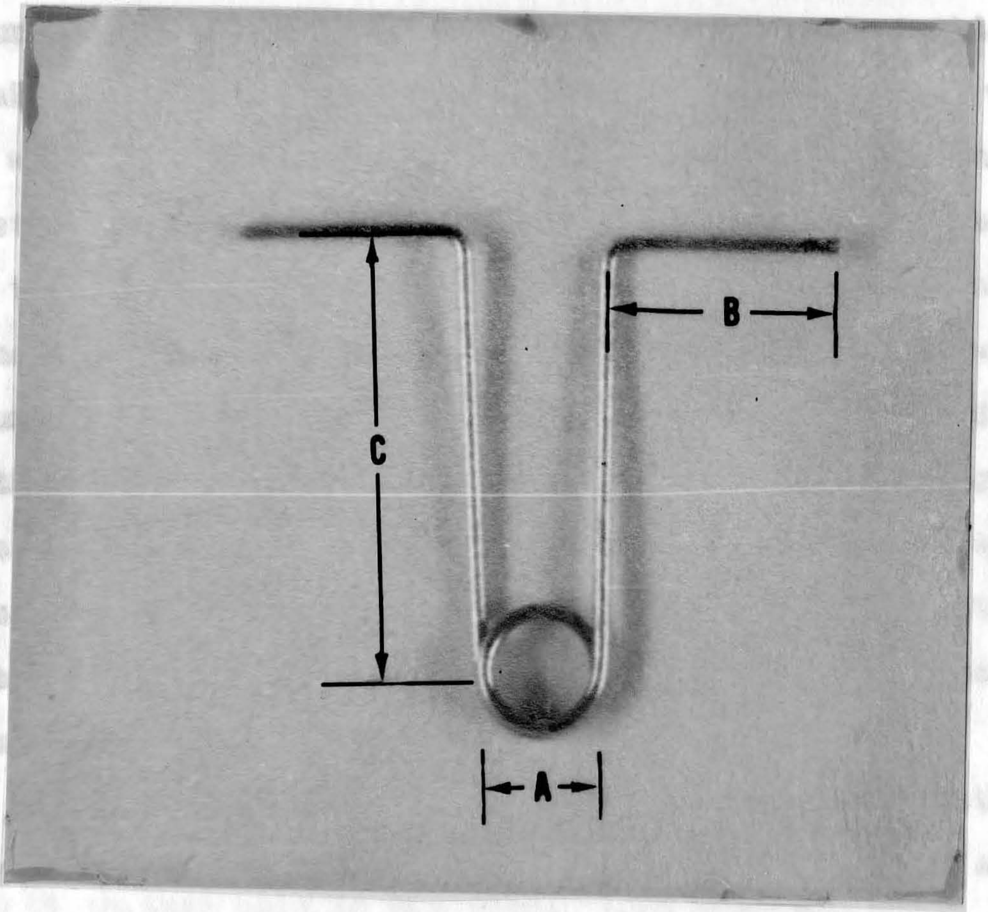

ELGILOY
Ultimate and yield stress, modulus of elasticity and percent elongation in area vs. percent reduction in area for as drawn and heat treated .025 diameter wire.
U.T. AS DRAWN
U.T. HEAT TREAT
Y.S. AS DRAWN
Y.S. HEAT TREAT
"E" AS DRAWN
"E" HEAT TREAT
% ELONGATION AS DRAWN
% ELONGATION HEAT TREAT
ULTIMATE HEAT TREATMENT
YIELD HEAT TREAT
ULTIMATE AS DRAWN
YIELD AS DRAWN
"E" AS DRAWN
STRESS X 10³ P.S.I.
PERCENT ELONGATION
MODULUS OF ELASTICITY E X 10⁶ P.S.I.
PERCENT REDUCTION IN AREA
0
10
20
30
40
50
60
70
80
90
100
200
300
400

Figure 2.6



A Diameter of Helix
B Spring Arm
C Spring Leg

Figure 2.7

Products Company. The bending of a specimen was started systematically at one end of the pre-cut wire. The first step was to make a right angle bend 5 mm. from the end, forming one of the arms. The desired length for the specimen leg was then measured on the wire and the desired bend started at this point. In making a helix in the specimen, a loop-bending plier was used. This was a three-stage plier capable of making helices having diameters of 1.5 mm., 2.0 mm., and 3.0 mm. The helix was bent into the specimen so that the wire emerging from the final turn of the helix was parallel to the first leg. This wire was then measured equal to the first leg and the right angle bend made for the second arm. The 5 mm. was measured for the length of the second arm and the wire cut at this point. The scrap of excess wire was then measured and its value was subtracted from the original 150 mm. to give the exact amount of wire used in the specimen.

All of the specimens were prepared in this systematic manner and a duplicate of each one was made to privide a replication for the experiment. The results of testing the duplicated specimens provided an estimate of error encompassing the whole process of making and testing the specimens.

Heat Treating:

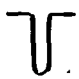
All the test specimens were heat treated over a gas flame until the wire turned a gun metal blue in color.

The various specimens have been arranged in tables which show the form of each specimen along with an identifying number and a list of the dimensions and other physical characteristics in Figure 2.8 A and Figure 2.8 B.

D. <u>Experimental Procedure:</u>

In order to test a simple ½-turn loop, Specimen No. 1, a clamp was mounted in the movable stage of the testing machine in such a position that it would hold a pin vise axially aligned with the pin vise on the force gauge. The collets of the pin vises were facing each other so that the lateral arms of the test specimen could be inserted into them when the specimen was to be tested with its arms restrained. Special holding devices or adapters were inserted into the pin vises for holding the specimen when it was tested with the arms free or unrestrained.

The test specimen which was to have its arms restrained was first placed into the pin vise on the force gauge with 4 mm. of the 5 mm. arm held in the vise. This vise was tightened. The movable stage was then moved closer until the opposite arm of the test specimen had entered the movable pin vise a distance of 4 mm. and this latter pin vise was tightened. Care was taken to be sure that the specimen was not under any force at the time

	TEST SPECIMEN NUMBER	NUMBER OF LOOPS IN THE SPECIMEN	NUMBER OF TURNS OF THE HELIX	DIAMETER OF THE HELIX IN (mm)	LENGTH OF LEG IN (mm)	TOTAL LENGTH OF WIRE IN (mm)
	1	½	½	1.5	7	25.5
	2	½	½	1.5	10	32.5
	3	½	½	3	7	27.5
	4	½	½	3	10	33.0
	5	1½	1½	1.5	7	30.0
	6	1	1½	3	10	38.0
	7	1	1½	3	7	35.5
	8	1	1½	3	10	41.5
	9	1	2½	1.	7	36.5

Test Specimens

Figure 2.8 A

	TEST SPECIMEN NUMBER	NUMBER OF LOOPS IN THE SPECIMEN	NUMBER OF TURNS OF THE HELIX	DIAMETER OF THE HELIX IN (mm)	LENGTH OF LEG IN (mm)	TOTAL LENGTH OF WIRE IN (mm)
	10	1	2½	1.5	10	42.5
	11	1	2½	3	7	44.0
	12	1	2½	3	10	48.0
	13	1	½	3	10	39.5
	14	1	1½	3	10	48.5
	15	1	2½	3	10	56.0
	16	3	1½ 1½ ½	3	10	43.0
	17	3	1½ 1½ 1½	3	10	52.0
	18	3	1½ 1½ 2½	3	10	60.0

Test Specimens

Figure 2.8 B

when it was initially mounted. Figure 2.11.

Now the force in grams exerted upon the specimen as a result of future compression or extension could be measured on the Hunter force gauge. The distance traveled by the movable stage in causing either compression or extension of the test specimen could be read directly on the Scherr-Tumico dial indicator.

The movable stage was brought closer to the fixed stage until the two legs of the specimen were nearly touching. Figure 2.12. A strip of .003" paper was used as a thickness gauge to determine the minimum spacing. Figure 2.13. At this time the dial indicator was set at a "zero" reference. Deflections could be measured in either direction from this point when the test specimen required it. The deflection of the force gauge was noted and recorded for this compressed condition of the specimen.

By withdrawing the movable stage from the fixed stage, the force on the test specimen was allowed to relax in uniform increments. At each division on the force gauge, the dial indicator was read and hence the distance which the specimen had expanded or returned was measured.

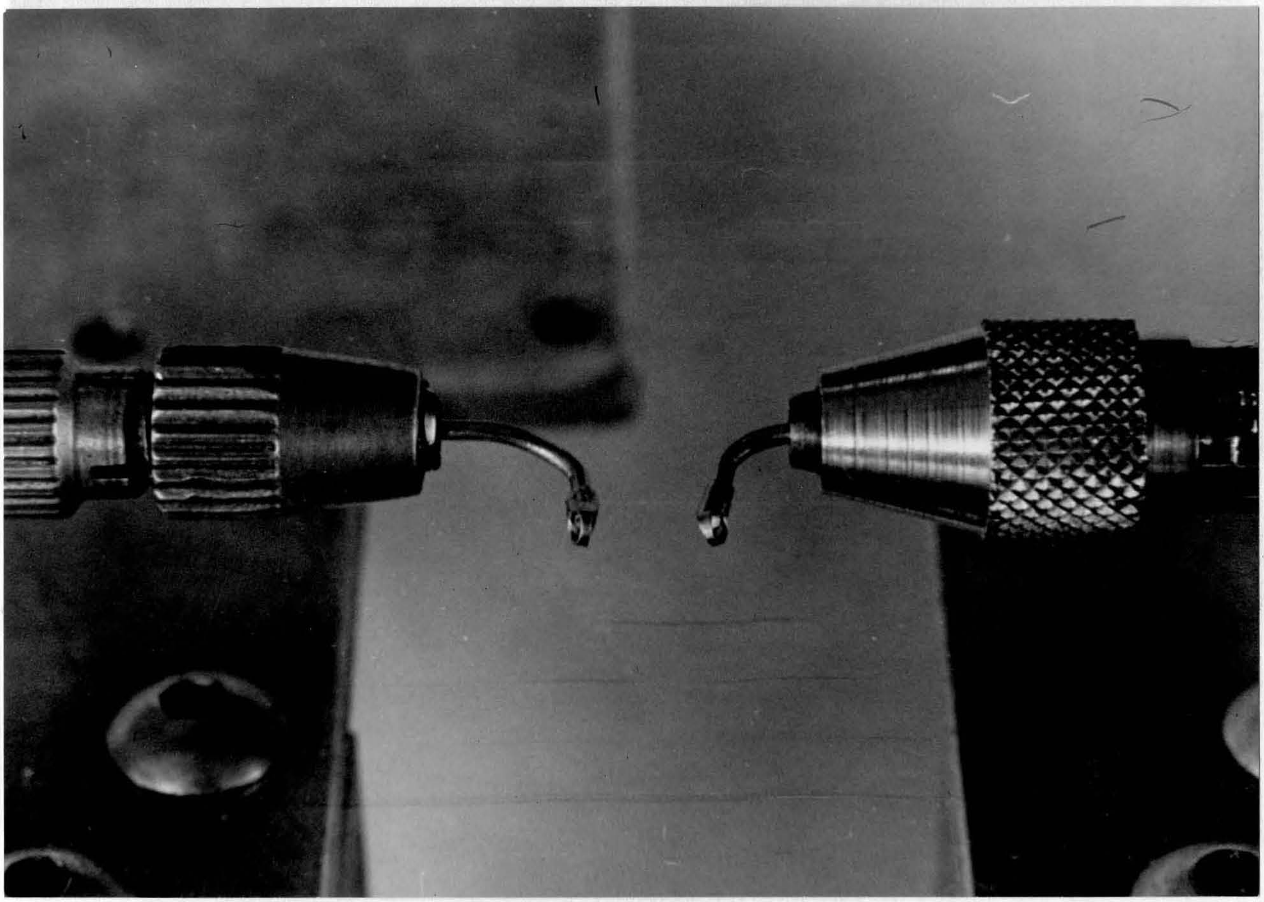
This process was continued until the force gauge returned to zero, indicating that the specimen had completely relaxed. Essentially this routine was followed in testing all of the specimens when the arms were restrained.

Some of the specimens were tested in a "free" system with the arms unrestrained. In order to do this, an adapter made out of .036" diameter stainless steel wire was prepared for each pin vise. Figure 2.9. These adapters were inserted in the pin vises and their eyelets were accurately aligned. A given test specimen such as No. 4 was mounted in the testing machine simply by having its arms inserted in the two eyelets as in Figure 2.10.

The movable stage was brought closer to the fixed stage to compress the test specimen until the two legs were nearly touching. A thickness gauge consisting of a strip of .003" paper was used to measure and set the minimum spacing of the legs. Following this the movable stage was withdrawn slowly from the fixed stage and the routine used with the restrained arms was repeated.

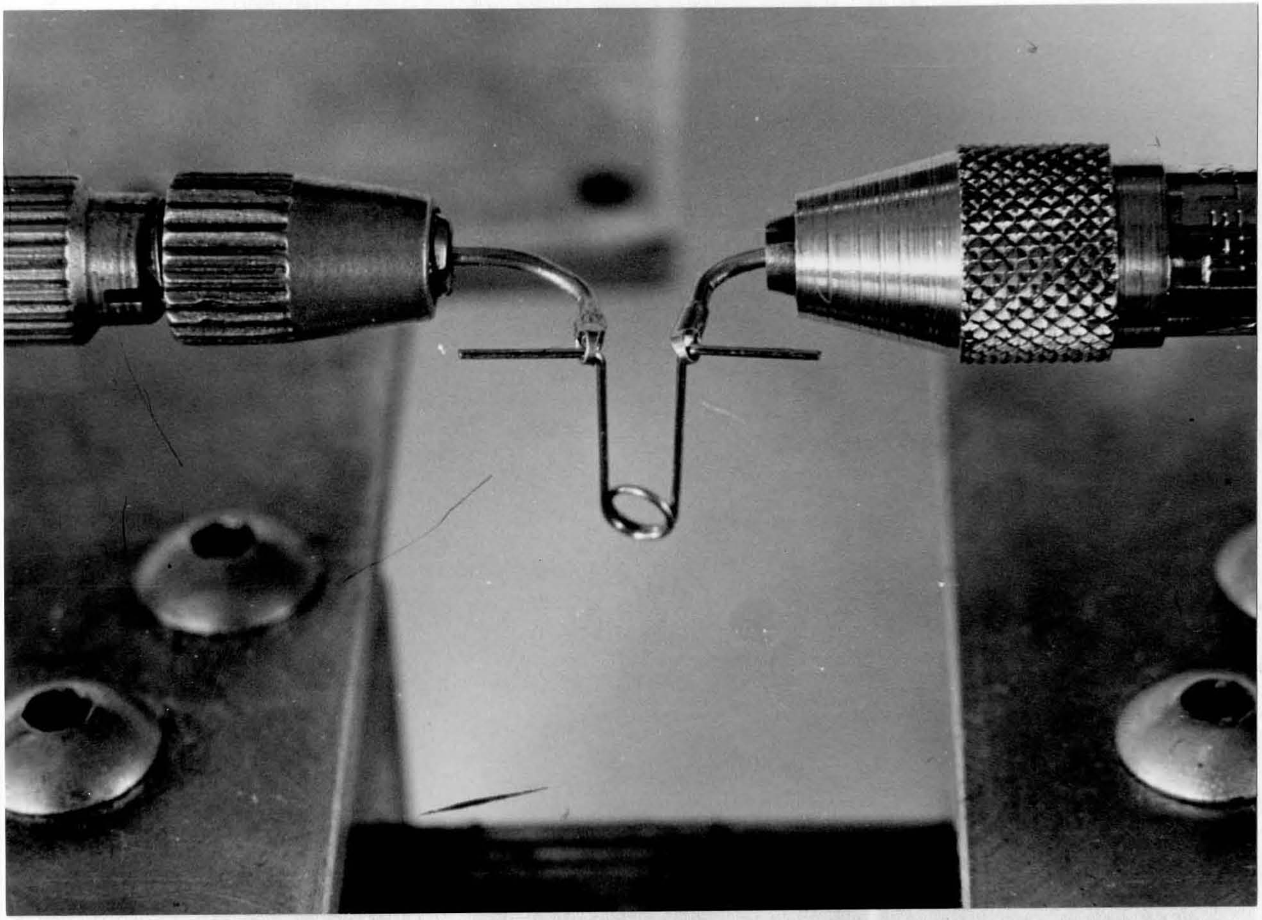
E. Experimental Design and Statistical Discipline:

Much of the work described in the literature dealt with the investigation of the elementary structural forms such as the cantilever beam, this beam with a helical loop located somewhere along its span, or a complete arch wire. Some of the work dealt with the specific spring formations such as the examples shown by Storey and Smith, but their study was not sufficiently concentrated on the mechanical details of any simple type of appliance to develop design principles.



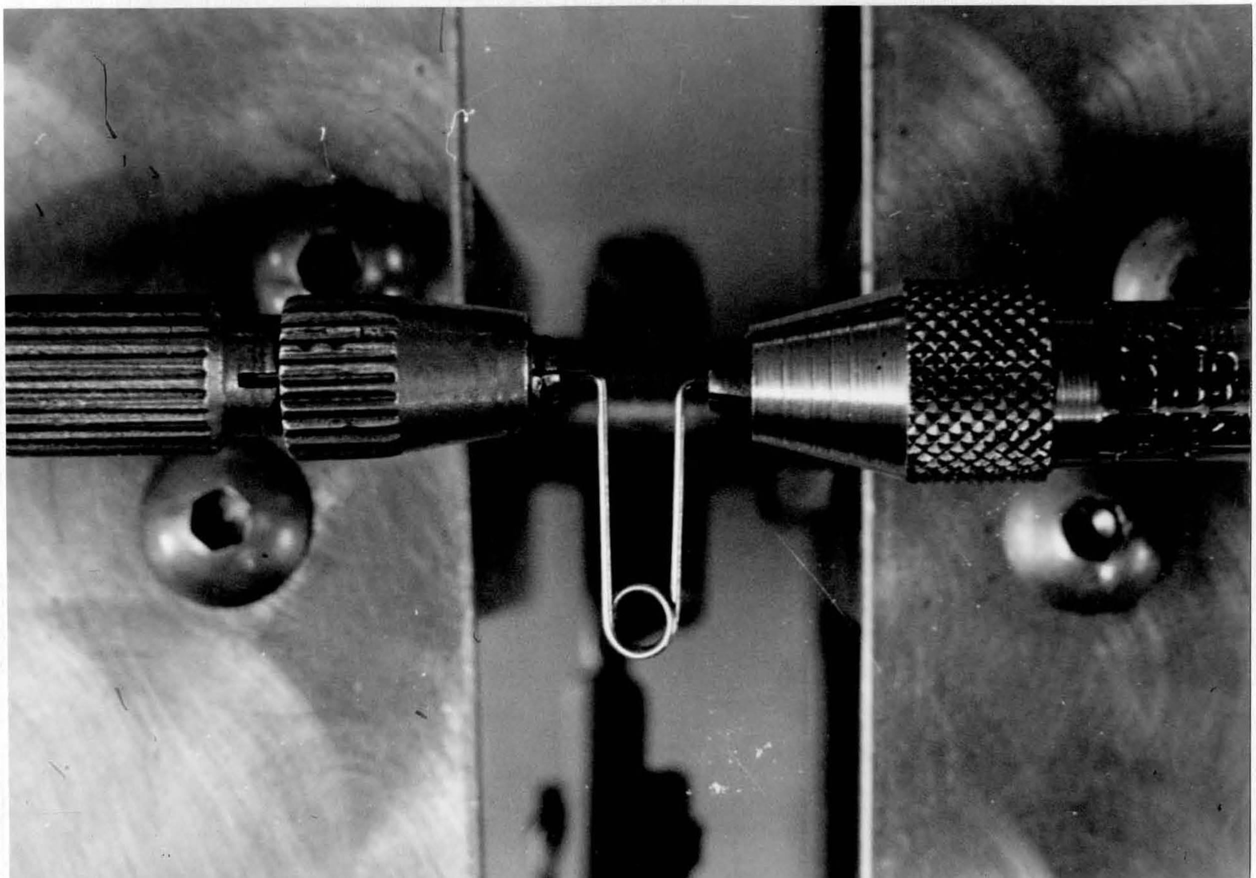
Adapter

Figure 2.9



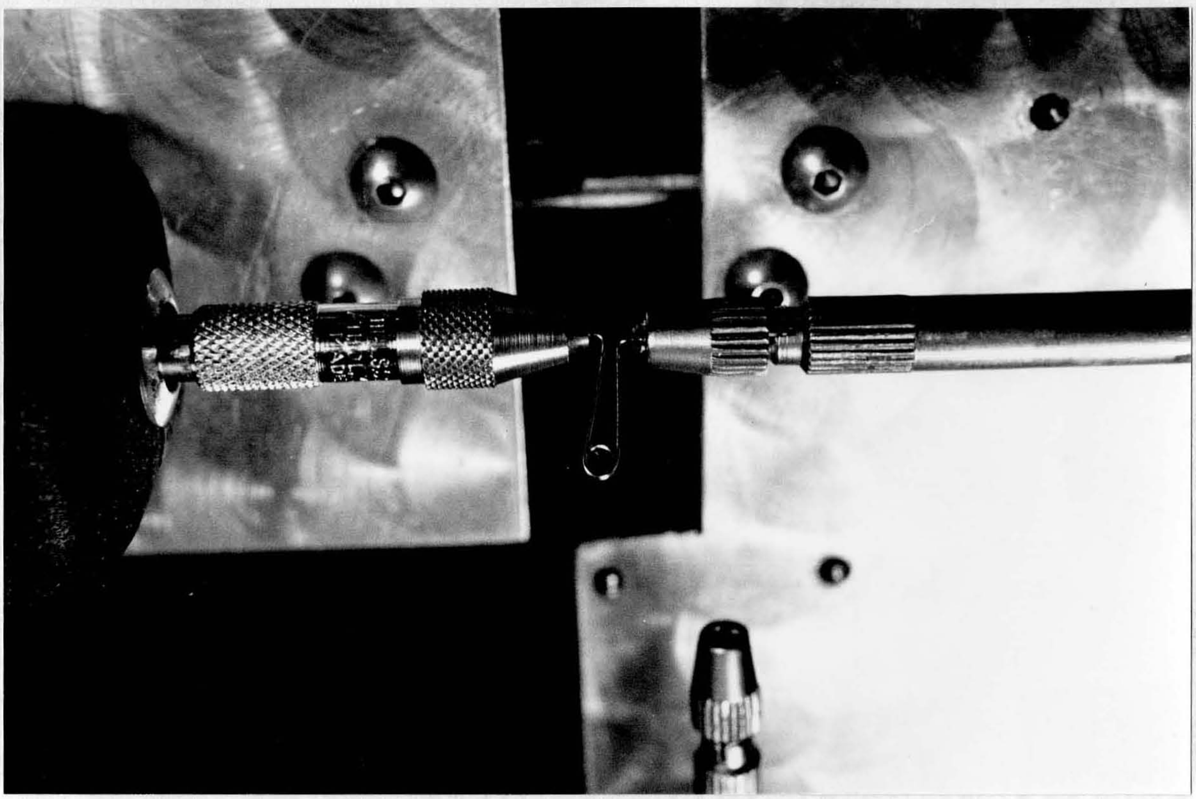
Mounted Specimen

Figure 2.10



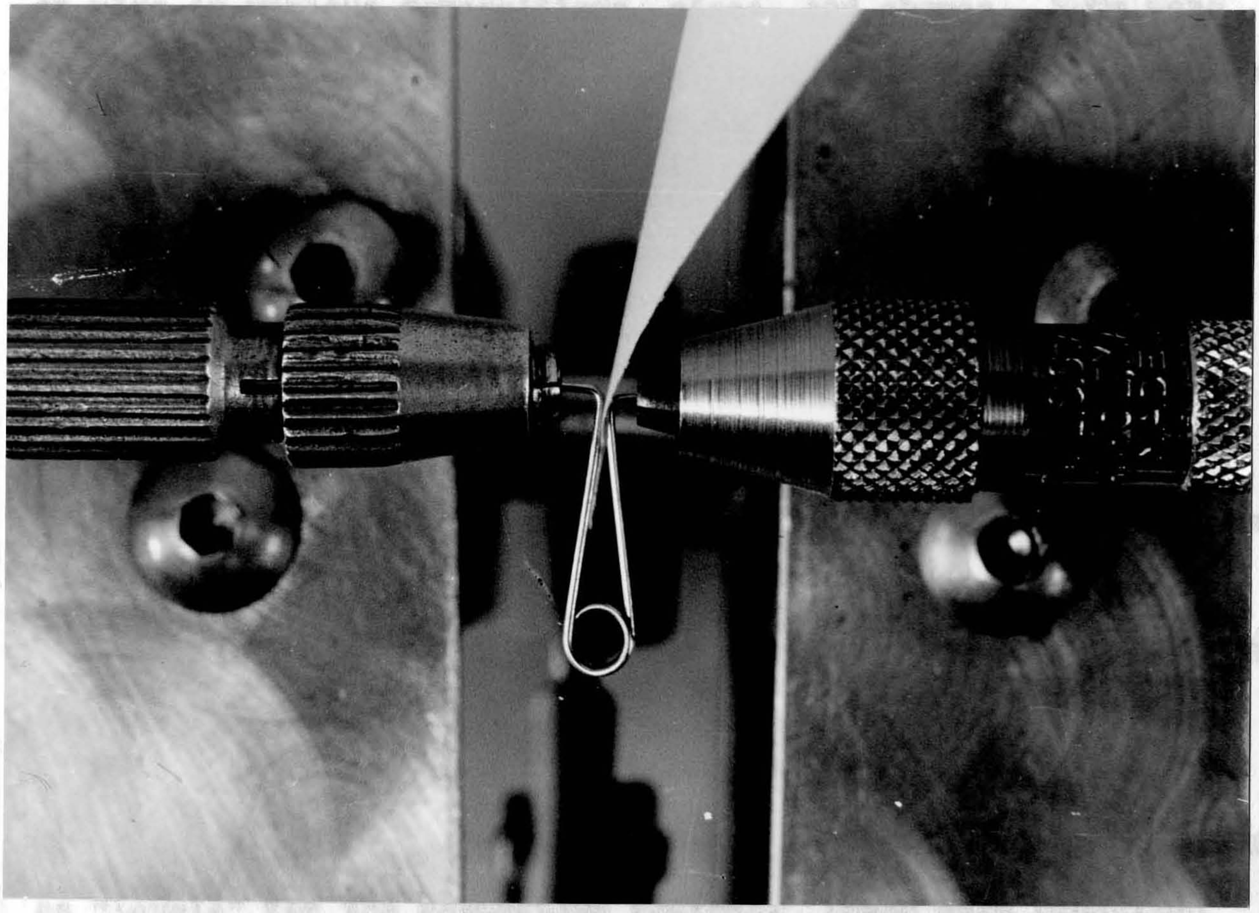
Mounted Specimen

Figure 2.11



Specimen Stressed

Figure 2.12



Thickness Gauge

Figure 2.13

In the early stages of this present study, various spring designs were constructed and methods of testing were investigated. Upwards of 108 various spring designs were fabricated of wire in various compositions, sizes, and shapes. Some were tested before heat-treatment and after heat treatment. The data from this work was plotted and appraised to learn the nature of the force-distance relationship.

Errors in testing techniques were recognized in the early pilot studies and the procedure was altered to reduce or eliminate these errors. From these efforts there developed a recognition of certain factors that were common to most designs. These factors were then used in planning the experiment for this study.

The three important factors in spring design were (1) number of turns in the helix, (2) the diameter of the helix, and (3) length of the legs. Loops of ½, 1½, and 2½ turns; diameters of 1.5 mm. and 3 mm., and lengths of 7 mm. and 10 mm. were selected as "levels" for the respective factors. These were organized into a 3x2x2 factorial design which required that springs embodying all possible combinations of these levels be formed and heat treated and tested in duplicate.

The chart shown in Figure 2.14 is a common one used in factorial designs of experiments. The sources of variation are

½ Turn				1½ Turn				2½ Turn			
1.5 Radius		3.0 Radius		1.5 Radius		3.0 Radius		1.5 Radius		3.0 Radius	
7 Leg	10 Leg	7 Leg	10 Leg	7 Leg	10 Leg	7 Leg	10 Leg	7 Leg	10 Leg	7 Leg	10 Leg

Design Chart

Figure 3.14

identified at each level and each one of the boxes along the bottom of the chart indicates one specimen. The first box corresponds with Specimen No. 1 while the last box corresponds with Specimen No. 12. There is a conventional analysis of variance scheme which was used to analyze the data obtained in this experimental design.

All of the springs specified in the design just described were "opening" springs but it was important to test some "closing" springs that were made with comparable dimensions. To accomplish this, springs of the "closing" type having dimensions like Specimens 4, 8, and 12 were constructed and designated as Specimens 13, 14, and 15. These six specimens were then compared in an appropriate analysis of variance table to determine whether there was any significant difference in spring rates due to the different direction of stressing.

Another series of specimens having dimensions comparable with those of Specimens 4, 8, and 12 was made up with lateral extension loops located at the right angle bends where the arms were formed. The purpose of this was to allow a comparison between these Specimens 16, 17, and 18, and Specimens 4, 8, and 12 which were tested with their arms free. The data were analyzed by a suitable analysis of variance schema to determine significance of differences between spring rates.

The foregoing statistical designs were chosen to provide for the study of interactions between the main sources of variation. It was known in advance that the main sources would be significantly effective in altering the spring rates but the interactions between such factors as turns in the helix and diameter of the helix cannot be predicted. These designs were intended to reveal interactions if they existed. The .05 level of probability is the highest one at which significance would be recognized.

The quantities measured were deflection in millimeters and force in grams. From these, spring rates were calculated and analyzed. The spring rate is defined as the change in force per unit change in deflection or Spring rate = $\frac{\Delta F}{\Delta S}$.

CHAPTER III

A. Experimental Results:

The results of the measurements described in Chapter II as made on Specimen No. 1 are shown in Figure 3.1. These have been corrected for the travel of the force gauge plunger. Reduction of the data in any given table to a single figure representing the spring rate is accomplished by means of a statistical method based upon the "least squares" principle of minimizing the residual error. The simplest application of this method is made when the line representing the data passes through the origin. Therefore, deflections in the table mentioned above were "inverted" to show just the distance traveled by the movable arm of the spring with respect to the arm attached to the force gauge. The resulting data are shown in Figure 3.2 A and are plotted in Figure 3.3.

The slope of the line is the ratio, $\frac{\Delta F}{\Delta S}$, which is the spring rate for Specimen No. 1. Since the various points defining the line are distributed along both sides of the line, it is necessary to use an "averaging" method to obtain the most reliable estimate of the slope of the line. This is given in the following formula $\quad SLOPE = \frac{FxS}{S^2}$

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)
00	1.758
10	1.708
20	1.673
30	1.618
40	1.573
50	1.523
60	1.488
70	1.443
80	1.403
90	1.363
100	1.316
110	1.268
120	1.221
130	1.173
140	1.133
150	1.083
160	1.038
170	.993
180	.933
190	.898
200	.858
210	.813
220	.768
230	.723
240	.693
250	.658
260	.608
270	.568
280	.523
290	.473
300	.423
310	.378
320	.323
330	.288
340	.248
350	.213
360	.168
370	.123
380	.078
390	.028
395	.000

Figure 3.1

Specimen Number 1

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.050	.045
20	.085	.085
30	.140	.135
40	.185	.180
50	.230	.210
60	.270	.255
70	.315	.305
80	.355	.335
90	.395	.395
100	.442	.457
110	.490	.490
120	.537	.537
130	.585	.575
140	.625	.625
150	.675	.680
160	.720	.720
170	.765	.780
180	.825	.820
190	.860	.865
200	.900	.910
210-	.945	.950
220	.990	.995
230	1.035	1.030
240	1.065	1.070
250	1.100	1.115
260	1.150	1.155
270	1.190	1.195
280	1.235	1.245
290	1.285	1.295
300	1.335	1.390
310	1.380	1.395
320	1.435	1.435
330	1.470	1.480
340	1.510	1.515
350	1.545	1.560
360	1.590	1.605
370	1.635	1.645
380	1.680	1.700
390	1.730	1.735
395	1.758	
400		1.780
405		1.782

Figure 3.2 A

Specimen Number 1

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (IN. RETURNED)	
	FIRST REPITITION	SECOND REPITITION
0	.000	.000
10	.125	.110
20	.220	.230
30	.355	.360
40	.485	.500
50	.605	.625
60	.710	.740
70	.810	.875
80	.930	.975
90	1.050	1.115
100	1.182	1.237
110	1.290	1.355
120	1.507	1.467
130	1.530	1.565
140	1.640	1.675
150	1.745	1.805
160	1.860	1.915
170	1.990	2.035
175		2.075
180	2.105	
190	2.220	

Figure 3.2 B

Specimen Number 2

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION ([illegible] RETURNED) FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.025	.100
20	.075	.065
30	.125	.125
40	.185	.170
50	.245	.225
60	.310	.290
70	.375	.340
80	.430	.390
90	.480	.455
100	.547	.517
110	.615	.580
120	.682	.642
130	.745	.695
140	.805	.765
150	.860	.825
160	.925	.890
170	1.000	.950
180	1.065	1.025
190	1.125	1.080
200	1.170	1.135
210	1.250	1.190
220	1.255	1.250
230	1.360	1.320
240	1.420	1.375
250	1.475	1.430
260	1.540	1.490
270	1.605	1.550
280	1.665	1.615
290	1.730	1.675
300	1.785	1.730
310	1.865	1.805
320	1.940	1.855
330	1.995	1.905
340	2.045	1.960
350	2.110	2.045
360	2.185	2.110
370	2.250	2.175
380	2.300	2.245
390	2.370	2.295
400	2.440	2.355
410	2.510	2.415
420	2.590	2.490
430	2.645	2.560
440		2.640
450	2.735	
460	2.800	
465	2.870	
470	2.960	
475	3.000	2.000

Figure 3.2 C
Specimen Number 3

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED) FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.130	.120
20	.275	.250
30	.440	.405
40	.585	.530
50	.715	.680
60	.835	.795
70	.965	.930
80	1.110	1.090
90	1.250	
100	1.407	1.352
110	1.535	1.500
120	1.677	1.647
130	1.795	1.775
140	1.895	1.900
150	2.050	2.030
160	2.180	2.150
170	2.310	2.280
180	2.440	2.410
190	2.565	2.535
195	2.698	
200		2.640
210		2.750
220		2.910

Figure 3.2 D

Specimen Number 4

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED) FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.005	.050
20	.110	.100
30	.180	.165
40	.235	.210
50	.295	.275
60	.345	.320
70	.395	.395
80	.455	.435
90	.525	.500
100	.597	.567
110	.660	.630
120	.727	.687
130	.780	.750
140	.835	.795
150	.895	.845
160	.950	.905
170	1.015	.985
180	1.090	1.050
190	1.150	1.105
200	1.215	1.160
210	1.270	1.215
220	1.330	1.270
230	1.395	1.330
240	1.450	1.385
250	1.495	1.435
260	1.550	1.490
270	1.595	1.550
280	1.625	1.595
290		1.655
300		1.690

Figure 3.2E

Specimen Number 5

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION ([illegible] RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.155	.165
20	.320	.325
30	.485	.485
40	.635	.650
50	.805	.830
60	.960	.980
70	1.130	1.110
80	1.270	1.240
90	1.420	1.390
100	1.602	1.564
105	1.745	
110		1.700
120	1.892	1.907

Figure 3.2 F

Specimen Number 6

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED) FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.065	.060
20	.145	.135
30	.225	.210
40	.305	.285
50	.390	.370
60	.470	.435
70	.550	.510
80	.610	.565
90	.695	.650
100	.792	.732
110	.875	.805
120	.952	.867
130	1.025	.940
140	1.105	1.020
150	1.190	1.085
160	1.265	1.170
170	1.355	1.250
180	1.450	1.335
190	1.530	1.400
200	1.600	1.470
210	1.670	1.545
220	1.755	1.620
230	1.840	1.685
240	1.920	1.765
250	1.970	1.820
260	2.045	1.890
270	2.130	1.975
280	2.220	2.045
290	2.305	2.120
300	2.375	2.195
310	2.450	2.280
320	2.530	2.355
330	2.605	2.425
340	2.685	2.495
350	2.750	2.560
360	2.840	2.650
370	2.905	2.720
375	2.932	2.767

Figure 3.2 G

Specimen Number 7

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.150	.147
20	.365	.337
30	.575	.542
40	.765	.752
50	.940	.967
60	1.110	1.157
70	1.345	1.382
80	1.520	1.542
90	1.715	1.752
100	1.897	1.929
110	2.070	2.132
120	2.342	2.324
130	2.525	2.507
140	2.705	2.692
150	2.880	2.877
160	3.045	3.052
165		3.180
170	3.245	
180	3.425	
185	3.523	

Figure 3.2 H

Specimen Number 8

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.065	.070
20	.140	.135
30	.225	.210
40	.295	.285
50	.375	.375
60	.445	.430
70	.520	.520
80	.765	.580
90	.665	.660
100	.742	.737
110	.840	.810
120	.907	.892
130	.990	.970
140	1.065	1.055
150	1.150	1.145
160	1.215	1.220
170	1.310	1.295
180	1.375	1.375
190	1.465	1.455
200	1.545	1.530
210	1.620	1.610
220	1.698	1.695
230	1.775	1.755
240	1.855	1.845
250	1.915	1.915
260	1.985	1.985
270	2.055	2.060
280	2.110	2.110
290	2.180	2.145
300		2.165

Figure 3.2 I
Specimen Number 9

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM [illegible])	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.205	.180
20	.360	.365
30	.560	.545
40	.735	.730
50	.940	.935
60	1.115	1.125
70	1.295	1.310
80	1.445	1.490
90	1.640	1.685
100	1.842	1.977
110	1.960	2.040
115	2.053	

Figure 3.2 J

Specimen Number 10

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION ([illegible]) FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.090	.080
20	.185	.150
30	.285	.230
40	.375	.325
50	.485	.420
60	.560	.495
70	.645	.600
80	.725	.675
90	.820	.765
100	.912	.867
110	.995	.965
120	1.082	1.057
130	1.165	1.150
140	1.250	1.230
150	1.345	1.335
160	1.425	1.405
170	1.530	1.520
180	1.620	1.615
190	1.715	1.720
200	1.795	1.805
210	1.880	1.895
220	1.970	1.980
230	2.045	2.055
240	2.155	2.155
250	2.240	2.235
260	2.320	2.325
270	2.410	2.430
280	2.590	2.520
290	2.575	2.625
300	2.660	2.710
310	2.750	2.805
320	2.820	2.880
330	2.885	2.950
340	2.940	
350	2.975	
355	3.018	

Figure 3.2 K

Specimen Number 11

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION [illegible] RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.175	.225
20	.390	.445
30	.620	.640
40	.815	.880
50	1.105	1.090
60	1.200	1.305
70	1.450	1.520
80	1.625	1.720
90	1.860	1.950
100	2.082	2.142
110	2.275	2.345
115		2.469
120	2.462	
130	2.695	
140	2.865	

Figure 3.2 L

Specimen Number 12

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.185	.195
20	.430	.435
30	.620	.620
40	.815	.840
50	1.025	1.015
60	1.230	1.215
70	1.460	1.430
80	1.665	1.635
90	1.910	1.830
100	2.117	1.982
110	2.300	2.180
120	2.532	2.387
130	2.760	2.550
140	2.990	2.785
150		2.960
152		3.028

Figure 3.2 M

Specimen Number 13

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION [redacted] RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.218	.325
20	.523.	.575
30	.853	.840
40	1.108	1.075
50	1.373	1.305
60	1.623	1.575
70	1.858	1.850
80	2.138	2.090
90	2.398	2.345
100	2.715	2.577
102	2.763	
110		2.805
112		3.033

Figure 3.2 N

Specimen Number 14

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.330	.375
20	.655	.620
30	.960	.945
40	1.245	1.235
50	1.510	1.460
60	1.865	1.775
70	2.185	2.030
80	2.470	2.350
90	2.845	2.635

Figure 3.2 0

Specimen Number 15

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.305	.155
20	.470	.325
30	.645	.490
40	.820	.660
50	.990	.850
60	1.135	1.005
70	1.305	1.165
80	1.475	1.325
90	1.665	1.520
100	1.827	1.682
110	1.990	1.865
120	2.157	2.047
130	2.325	2.225
140	2.480	2.385
150	2.800	2.555
160	3.070	2.730
170		2.900
180		3.085
190		3.220

Figure 3.2.P

Specimen Number 16

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM [illegible])	
	FIRST [illegible]EPETITION	SECOND REPETITION
00	.000	.020
10	.185	.230
20	.410	.435
30	.665	.660
40	.880	.925
50	1.115	1.180
60	1.320	1.405
70	1.590	1.645
80	1.820	1.8[illegible]5
90	2.100	2.125
100	2.327	2.352
110	2.565	2.565
120	2.712	2.837
130		3.000

Figure 3.2 q

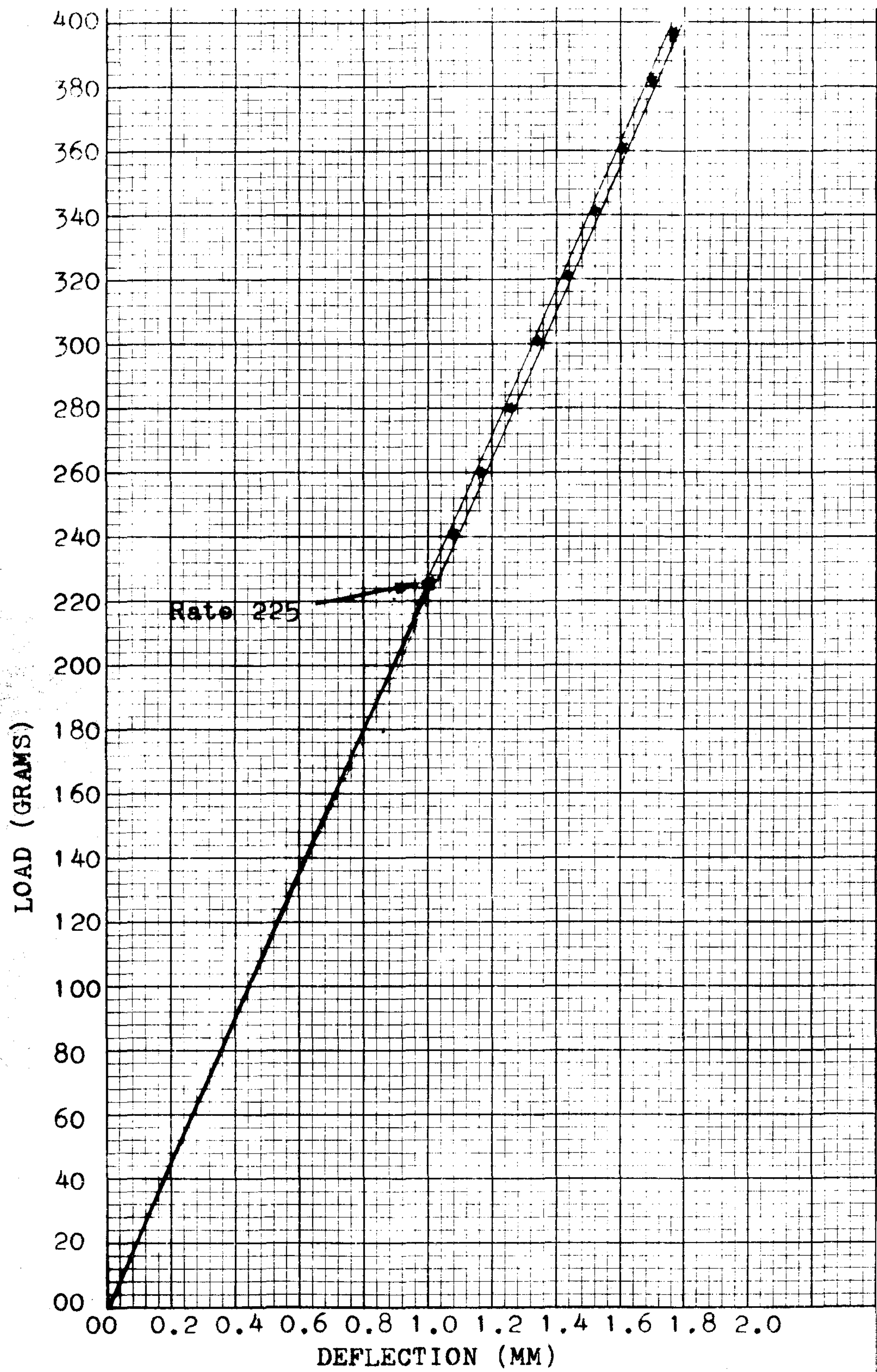
Specimen Number 17

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.260	.255
20	.555	.560
30	.880	.860
40	1.175	1.245
50	1.465	1.445
60	1.710	1.715
70	2.005	2.005
80	2.310	2.255
90	2.545	2.565
100	2.852	2.877
105	2.896	
110		2.916

Figure 3.2 R

Specimen Number 18



Load-Deflection Graph Specimen No. 1
Figure 3.3

All of the specimens tested yielded spring rates which have been tabulated in their respective tables for further analysis. The specimens belonging to the factorial design are listed in Figure 3.4. Specimens in Figure 3.5 are various forms of the "closing" loops. Specimens numbered 16, 17, and 18 are constructed with lateral loops at the right angle bends where great stress was created in the analagous Specimens numbered 4, 8, and 12. The results of tests upon these specimens with the lateral loops are tabulated in Figure 3.6 A along with results of testing Specimens numbered 4, 8, and 12 in the "free" or unrestrained system in Figures 3.6 B, B4, C8, D12.

The table of spring rates for the factorial design was analyzed in accordance with the original plan for this design. It was necessary to transform the values by the square root transformation to obtain the required homogeneity of variance. Analysis of Variance Table Figure 3.7 shows the results of this work.

From the 12 degrees of freedom available for estimating experimental error, it was found that the standard deviation of experimental error was .1296 units. Since the average entry in the transformed data sheet Figure 3.4 was 10.118, this means that the coefficient of variation was .0128 or about 1.28 per-cent. This was regarded as being reasonable and the interactions

TABLE OF SPRING RATES

Specimen Number	Rates Initial	Rates Replication	$\sqrt{\text{Rates}}$ Initial	$\sqrt{\text{Rates}}$ Replication
1	225.33	219.96	15.01	14.83
2	85.07	81.17	9.22	9.01
3	167.88	164.86	12.95	12.84
4	74.25	72.91	8.62	8.54
5	174.05	166.97	13.19	12.92
6	63.74	62.65	7.98	7.91
7	136.29	126.28	11.67	11.23
8	52.20	52.08	7.22	7.22
9	131.18	130.72	11.45	11.43
10	54.41	52.76	7.38	7.26
11	112.37	111.68	10.60	10.57
12	48.38	46.41	6.96	6.81

Figure 3.4

Opening Loop

TABLE OF SPRING RATES

Specimen Number	Rates Initial	Rates Replication	$\sqrt{\text{Rates}}$ Initial	$\sqrt{\text{Rates}}$ Replication
13	49.96	45.21	7.07	6.65
14	38.17	37.11	6.18	6.09
15	36.92	32.22	6.08	5.68

Figure 3.5

Closing Loop

TABLE OF SPRING RATES

Specimen Number	Rates Initial	Rates Replication	$\sqrt{\text{Rates}}$ Initial	$\sqrt{\text{Rates}}$ Replication
16	58.96	55.62	7.68	7.46
17	43.56	42.67	6.60	6.53
18	34.86	32.59	5.90	5.71

Figure 3.6 A

Lateral Extension Loop

TABLE OF SPRING RATES

Specimen Number	Rates Initial	Rates Replication	$\sqrt{\text{Rates}}$ Initial	$\sqrt{\text{Rates}}$ Replication
4	30.85	30.44	5.55	5.51
8	16.25	14.56	4.03	3.81
12	14.08	13.46	3.75	3.67

Figure 3.6 B

"Free" System

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
)	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.285	.307
20	.605	.602
30	.945	.947
40	1.310	1.282
50	1.665	1.637
60	1.960	1.942
70	2.33	2.292
75	2.462	
80		2.672
85		2.869

Specimen Number 4

"Fred" System

Figure 3.6 B

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.575	.395
20	1.200	1.920
30	1.805	2.470
40	2.515	2.920
48	3.102	
49		3.226

Specimen Number 8

"Free" System

Figure 3.6 C 8

TABLE OF LOAD DEFLECTION VALUES

LOAD (GRAMS)	DEFLECTION (MM RETURNED)	
	FIRST REPETITION	SECOND REPETITION
00	.000	.000
10	.665	.635
20	1.465	1.400
30	2.265	2.165
35		2.273

Specimen Number 12

"Free" System

Figure 3.6 D 12

were then tested for significance.

The three factor interaction (turns x diameter x length) was found to be significantly large when judged at the .01 level of probability. This was not surprising because of the complex relationship between the three variables. All of the two-factor interactions were significant when judged at the .001 level of significance. The relations between variables which gave rise to these interactions are depicted in the graphs of Figure 3.8. Part (A) in this illustration shows the lack of parallelism between two lines representing the two diameters of the helices. It is this lack of parallelism that is responsible for the large interaction mean square in Figure 3.7. In graph (B) of Figure 3.8 the lack of parallelism between the lines for T ½, T 1½, T 2½ demonstrated interaction which is borne out in the large interaction mean square found in Figure 3.7. In graph (C) of Figure 3.8 the lack of parallelism between the lines for T ½, T 1½, T 2½ illustrate again interaction that is confirmed when the large interaction mean square is found in Figure 3.7.

The three main effects, turns, diameter of loop and length of legs were all highly significant in their influence upon spring rates. The manner in which they are related will be discussed later.

ANOV TABLE

SOURCES OF VARIATION	D.F.	S.S.	M.S.	F	F.01	F.001
Turns	2	22.0096	11.0000	very large	6.93	12.97
Diameters	1	6.3654	6.3600	very large	9.33	18.64
Length	1	124.0330	124.0300	very large	9.33	18.64
TXD	2	.4583	.2290	13.63	6.93	12.97
TXL	2	1.3682	.6840	40.71	6.93	12.97
DXL	1	1.2979	1.2980	77.26	9.33	18.64
TXDXL	2	.2872	.1430	8.51	6.93	12.97
Replications	12	.2023	.0168			
Total	23	156.0219				

Figure 3.7

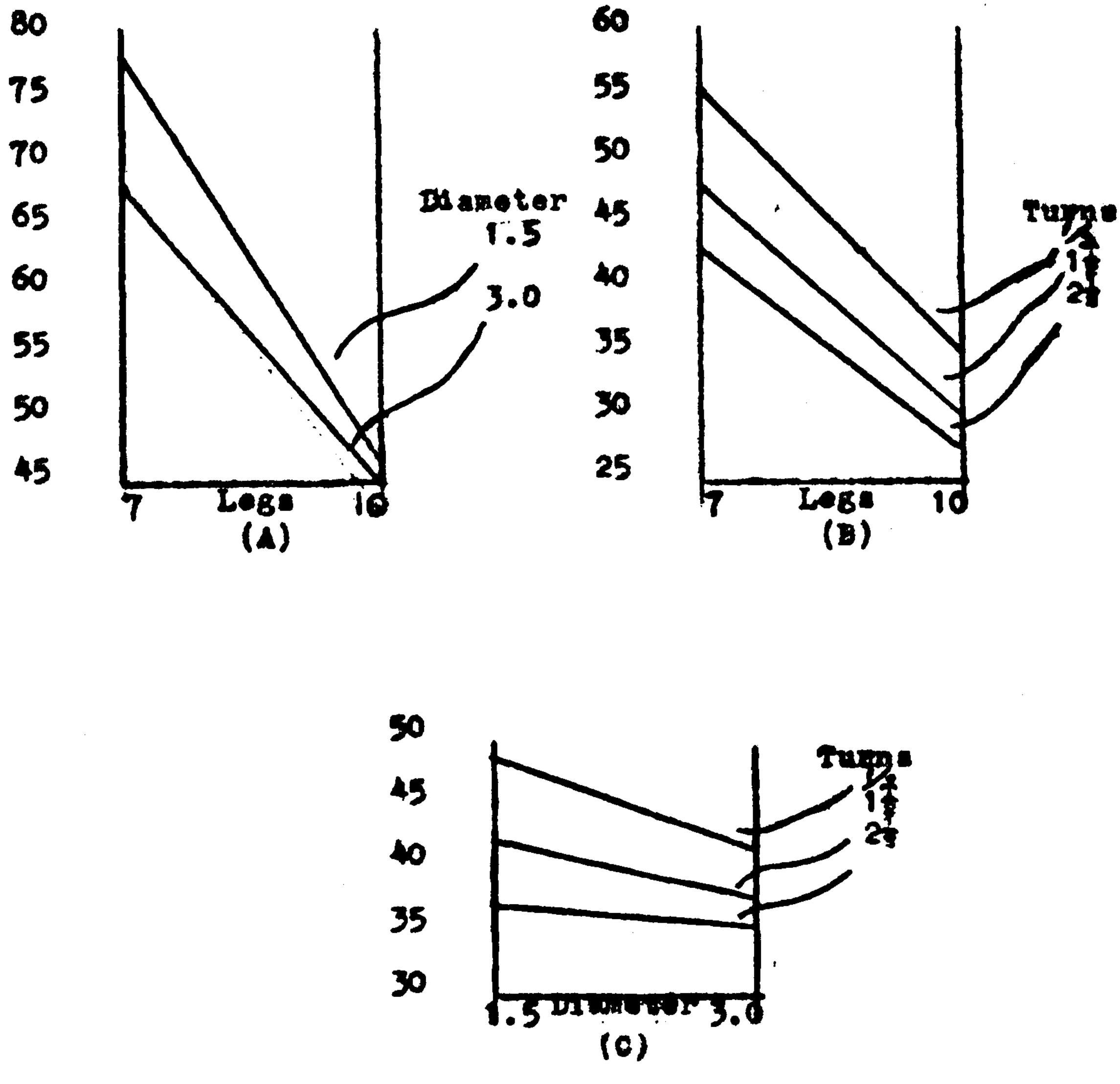


Figure 3.8

Graphs illustrating interactions

TRANSFORMED DATA TABLE

SPECIMEN NUMBER	FIRST REPLICATION	SECOND REPLICATION
<u>Closing</u>		
13	7.07	6.65
14	6.18	6.09
15	6.08	5.60
<u>Opening</u>		
4	8.62	8.54
8	7.22	7.22
12	6.96	6.81

Figure 3.9

ANOV TABLE

SOURCES OF VARIATION	D.F.	S.S.	M.S.	F.	F.05	F.01
Turns	2	3.9503	1.9751	63.51*	5.14	10.92
Direction	1	4.8387	4.8387	155.58*	5.99	13.74
TXD	2	.3070	.1535	4.935	5.14	10.92
Replication	6	.1867	.0311			
Total	11	9.2827				

Figure 3.10

Data in Figure 3.5 have been analyzed along with comparable data from Figure 3.4. The specimens being compared from the latter table are Nos. 4, 8, and 12. In Figure 3.9 there are tabulated the transformed values of spring rates from these specimens, while in Figure 3.10 there is the Analysis of Variance Table resulting from the analysis of these data. The coefficient of variation from this work is about 2.54 which is appreciably greater than the one from the factorial design but this is not so large as to render the work unusable.

There was no significance shown by the 2-factor interaction probably because of the large experimental error. Both "Turns" and "Directions" had a highly significant effect upon the spring rate in spite of the large experimental error. It was known from the prior work that "Turns" would be influential but the purpose of this comparison was to see the influence of the direction of action of the spring. A discussion of the reasons for this great difference between the two types of springs will be given later.

The results of comparing the specimens containing the lateral loops and the other specimens that were tested in the "free" and also the restrained system are analyzed in Figure 3.12. Here the standard error was only .0985 units and the coefficient of variation was .0159 or about 1.6% of the average

TRANSFORMED DATA TABLE

SPECIMEN NUMBER	FIRST REPLICATION	SECOND REPLICATION
Restricted System		
4	8.62	8.54
8	7.22	7.22
12	6.96	6.81
"Free" System		
4	5.55	5.51
8	4.03	3.81
12	3.75	3.67
Lateral Loop		
16	7.68	7.46
17	6.60	6.53
18	5.90	5.71

Figure 3.11

ANOV TABLE

SOURCES OF VARIATION	D.F.	S.S	M.S.	F.	F.05	F.01
Turns	2	10.0849	5.0424	Very Large	4.26	8.02
Forms	2	32.0508	16.0254	Very Large	4.26	8.02
TXF	4	.2394	.0598	6.16	3.63	6.42
Replications	9	.0874	.0097			
Total	17	42.4625				

Figure 3.12

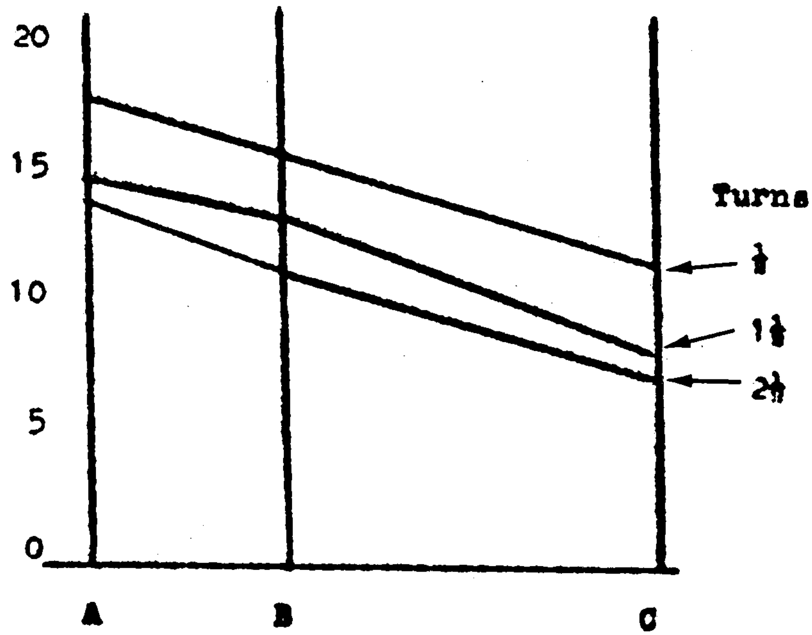
reading in Figure 3.11.

The interaction between "turns" and "forms" was highly significant as judged at the .01 level of significance. Figure 3.13 shows the lack of parallelism between the three lines that is responsible for the large interaction mean square. Obviously the springs containing the 1½-turn helix did not respond quite like the other specimens.

There are many interesting aspects of this test shown in Figure 3.13. The 1½-turn helices and the 2½-turn helices have similar spring rates that are appreciably lower than the rates of the ½-turn loops. The springs made with lateral extension loop had spring rates somewhat below those of the springs that had their arms restrained. These same springs when tested with their arms free had spring rates that were very much reduced. Figure 3.14.

Figure 3.14 shows the actual spring rates for the specimens tested and analyzed in Figure 3.12. Since only the transformed data were plotted in Figure 3.13, one could not read the actual spring rates in grams per millimeter and hence could not make comparisons. In the graph of Figure 3.15 a true picture of relative spring rates is shown. As an example of the comparisons possible, Specimen No. 4 when tested in the "free" system had a spring rate only 41.5% of the rate shown when tested with arms restrained.

INTERACTION GRAPH



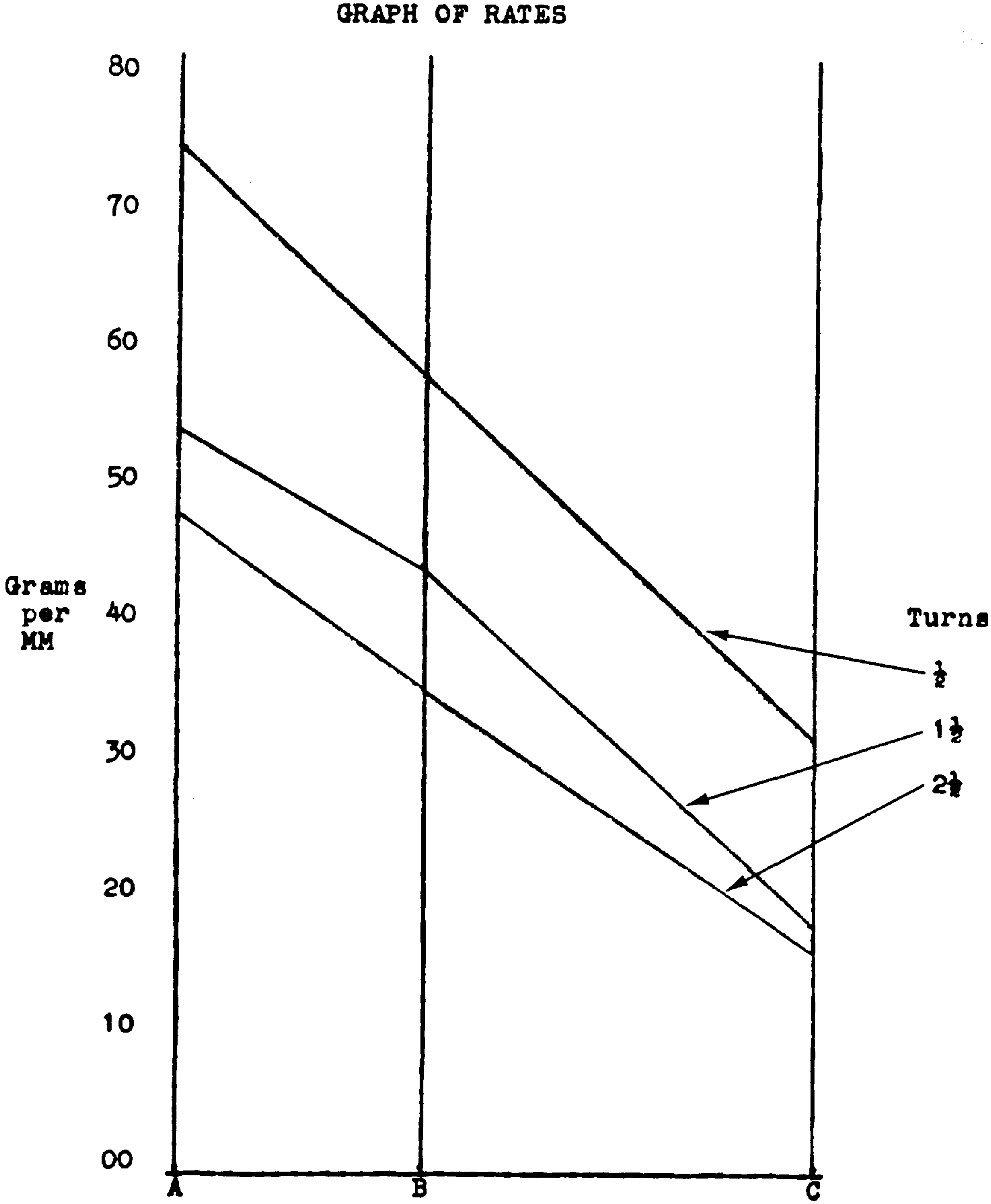
A Restricted System
B Lateral Loops
C "Free" System

Figure 3.13

SPRING RATES
(Average of two replications)

Number of Turns of Helix	Restricted Arms	Lateral Loop Helices	"Free" Arms
1/2	73.61	57.30	30.58
1 1/2	52.12	43.03	15.36
2 1/2	47.33	33.75	13.76

Figure 3.14



A Restricted System
B Lateral Loops
C "Free" System

Figure 3.15

DISCUSSION

It was the purpose of this study to measure and to analyze the force and deflection values of various designs of wire springs in order to learn the influence of spring characteristics upon the design of a practical orthodontic device. Clinically, the magnitude of a force and the distance through which the force acts are of significant value. Therefore they are very important in the design and application of an orthodontic appliance. Combining these two factors, it is appropriate to use the rate of force change per unit of deflection to appraise or to evaluate the design of an appliance. This rate is the "spring rate" which has previously been described and the spring rates for the numerous specimens that have been made and tested are considered to be the final results of the experimental work.

There has been a tendency for articles in the literature to generalize in describing the influence of such factors as length of the legs of a loop and the diameter of the helix without pointing out the fact that these dimensions of the spring interact. Figure 3.8 A shows that two specimens having legs 7 mm. long had greatly different spring rates because one of them was made with a helix having 3 mm. diameter while the other had a helix only 1.5 mm. in diameter. The larger diameter helix yielded a spring rate only 78.5% as high as the smaller diameter helix. When two specimens were constructed with legs 10 mm. long their spring rates differed only slightly even though one had a helix 3 mm. in diameter while the other had a helix 1.5 mm. in diameter. The larger diameter helix yielded a spring rate 86.6% as high as the smaller diameter helix. From this it must be clear that one cannot make a meaningful generalization about the effect of changing either the length of the legs or the diameter of the helix independently. There must be some more complex relation between these factors.

Burstone (1961) states that the insertion of a 3 mm. diameter loop having one full turn reduced the spring rate 46% below that of a straight cantilever beam 10 mm. in length. The insertion of a similar helix having 2 turns reduced the spring rate 58% below that of the straight beam. While these figures are undoubtedly correct, they do not indicate the fact that there is a significant interaction between the number of turns in the helix and length of the beam.

Figure 3.8 B shows that there was an interaction between the number of turns in the helix and the length of the legs in a helical loop spring. If the springs had legs 7 mm. long, then a 2½-turn helix yielded a spring rate that was 62.7% of the rate of the spring having just ½-turn. If the spring had legs 10 mm. long, the spring having a helix with 2½ turns had a spring rate that was 64.4% of the rate of the spring with the ½ turn.

If the diameter of the helix was 1½ mm., then changing from ½-turn to 2½-turns reduced the spring rate by 39.1%. If the diameter of the helix was 3 mm., then changing from ½-turn to 2½-turns in the helix reduced the spring rate 33.8%. It can be seen in Figure 3.8 C that the lack of parallelism between the three lines representing the various number of turns in the helices discloses this condition of interaction which is being described. It is unfortunate that in much of the work of the past, there has been an incomplete picture of the problems being encountered. Sometimes the engineering information available to the investigators was too deficient to be of much value. Other times it was not appropriate unto the mechanics of the problem.

Brumfield (1930) gave a formula which may be written as $D = \frac{C W l^2}{E d}$. The small d in the denominator is said to represent the depth of the section of the beam in the plane of bending. Waverley and Richardson (1947) gave a similar formula as follows: $D = \frac{C W l^3}{E I}$. In this, the I in the denominator represents the moment of inertia of the section of the beam around the axis in the neutral plane. Both of these are supposed to be applied to a cantilever beam, load by a concentrated force, W at a distance, l from the point of support of the beam. Clearly one of them must be wrong. The latter one agrees with the engineering texts dealing with beams.

Another source of unreliable information about the characteristics of the wires used by orthodontists is the classical type of experimentation that has been used. The usual method of varying one parameter at a time and observing the results of making such variation provides accurate information about the exact system that is being investigated. But when one wishes to extrapolate this knowledge to a slightly different situation, he may be gravely in error if there is an unknown interaction between several parameters. It is in an effort to avoid this common error that the factorial design of experiment was employed in this present study.

Another source of uncertainty in the understanding of published data is the lack of a measure of the experimental error which lies behind the quoted numerical data. In the performance of this experiment care was taken to include at all points a measure of the experimental error incurred in preparing the specimens and making the measurements. The mean of all of the spring rates determined in the factorial experiment was 102.4 grams per millimeter. The maximum error likely to be encountered in the measurement of any rate in this region is plus or minus one gram per millimeter. This estimate of error is derived from the whole factorial experiment and is completely objective. Each portion of this study was planned to yield its own measure of experimental error.

Hanbooks and published data sheets state values of modulus of elasticity and yield strength, etc. with the understanding that such values are subject to wide variation. This is generally recognized and it causes little concern, but published data resulting from experimental work are not generally interpreted in this manner.

When three closing loops were tested and analyzed against three comparable opening loops it was shown that a significant difference existed between the spring rates for the two kinds of loops. The average rate for the opening loops was 57.18 grams per mm. and the average rate for the closing loops was 39.58 grams per mm. The amount of wire used in the opening loop with ½-turn was 83.5% as much as was used in the comparable closing loop. But the spring rate for the opening spring was 56.4% higher than the rate for the closing spring.

As Burstone has stated, the distribution of the wire in a spring is more influential than the amount of wire in the spring. The placing of wire in the positions of greatest bending moment increases the allowable deflection without increasing the fiber stresses in the wire. The wire at the right angle bend is severely cold-worked and when it is heat-treated it becomes very stiff due to the manner in which Elgiloy responds to working and subsequent heat treating. This means that in the opening loops, where the wire extended only one mm. from the pinvise to the bend, there was very little beam length in which to flex. In the closing loops, there was a distance of four mm. from the pinvise to the right-angle bend. This gave enough length for flexing to provide a reduced spring rate. There appeared to be no significant interaction between turns in the helices and the direction of activation of the springs. This should mean that the number of turns in the helix would have no real effect on the change in spring rate between the two kinds of loops (opening vs. closing). In reality, the error mean square for this study was almost twice as large as it had been for the factorial experiment and if we use the smaller mean square as a more nearly valid estimate, we can see that the interaction term should be regarded as significant. The reason for the larger experimental error in this work was believed to be due to the friction between the two crossed arms of the closing loops. There was much greater difference between the rates of the two loops having the ½-turn loop than there was between the two loops having the 2-turn helix.

As a result of earlier work, it was expected that the placing of more wire in the arms of a helical loop would reduce the spring rate. It was therefore decided to include an experiment in which three degrees of restraint would be investigated. The completely restrained arms were to be compared with completely unrestrained arms and also with some specially constructed specimens having lateral extension loops placed at the points where the great cold-working had been done. The thought was that forming a 1¼-turn helical loop with a diameter of one mm. would create less cold working and less stiffness at the points where the right angle bends were made. When the data were analyzed it was found that the average spring rate for the restrained arm specimens was 57.12 grams per mm. and the average rate for these same specimens when tested with their arms free was only 19.24 grams per mm. - a reduction of 66.4%. The average rate for the lateral loop specimens was 44.18 grams per mm. which was only 77.2 percent of the spring rate for the completely restrained arm specimens. The inclusion of the loops reduced the spring rate 22.8 percent.

There was no doubt about there being interaction between the number of turns in the helix and the manner of holding the springs. The relations were depicted in Figure 3.15 where the data for the 1¼-turn loop contributes most to the interaction mean square. The factors which cause the departure from a straight line have not been isolated yet but the work has been repeated with the same results and therefore the departure has appeared to be a reproducible phenomenon.

The importance of the manner in which the arms of a helical loop are held has been stressed only infrequently in the literature. In most instances the manner in which the spring forms have been held cannot be accurately determined. In the work of Storey and Smith there is enough information to disclose the manner in which the specimens were held for testing. One of their methods was to hold one of the arms rigid and to apply concentrated force to the other arm without holding it rigidly. Burstone, in describing the testing of various springs, has mentioned the care used in holding the ends of the springs. This part of the experiment has demonstrated that the construction and holding of the arms of a helical loop are very important in determining the final spring rate.

In the U shaped spring which has been called the simple loop, there are mechanical elements which require complex applications of calculus in order to achieve a fairly complete analysis. The helical loop springs are even more difficult to analyze rigorously. To analyze in this sense, means to express mathematically the relations between the following properties of a spring: Length of legs, diameter of the helix, modulus of elasticity of the wire, applied forces, deflection of the legs, and moment of inertia of the cross-section of the wire.

The process of running load-deflection curves on all of the various forms of wire springs was necessary to provide data for the individual studies already described but its secondary purpose was to enable the investigator to check the validity of the mathematical formulae relating the various properties of springs. As mentioned in the review of literature other investigators have attempted some phases of this work but each time, the form of spring is changed in some critical respect and formulas are no longer applicable.

A formula quoted in Smith and Story (1952) might be applicable to the helical loops with slight modification. As written it presumably applied to a loop having one arm "free" and the other restrained. This formula can be written in terms of spring rate as follows: Spring rate $= \frac{F}{S} = \frac{1}{K}$

Where $K = \dfrac{128\, r\, L\, (\pi n\, L + r)}{\pi E d^4} + \dfrac{128\, L^3}{3\pi E d^4}$

L = the length of each arm
r = the radius of the coil
n = the number of turns
E = the modulus of elasticity of the material
d = the diameter of the wire

Using the spring rates measured from specimens Numbers 4, 8, and

12, tested in the free system, the reciprocal of K used by Smith and Storey has been calculated as shown in the following table. Using the formula given by Smith and Storey, corresponding values of 1/K have been calculated for comparison. All values are in English units and so the spring rates are in pounds per inch.

Specimen Number	1/K as Measured	1/K as Calculated
4	1.71	1.31
8	.86	.73
12	.77	.51

The value of E used in the calculations was 29.15 millions of psi, which is obtained from data supplied by the Elgin Watch Company.

Agreement between the measured rates and the calculated rates was not very good but when the comparison between spring formations is made it can be seen that Storey and Smith used helical loops which had one end completely free and the other end restrained. It is to be expected that a modification of the formula would be needed to account for this difference.

A formula for the inclusion of a helical loop in a cantilever beam such as might be employed in a reflex rotation lever was given by Burstone (1961).

$$\text{Spring rate} = \frac{F}{S} = \frac{EI}{(2\pi r n) \quad (L^2 \frac{r}{2}\, 2) + 4Lr^2 \frac{L}{3}^{3}}$$

It was not possible to fit any of the data from this present study into this formula and achieve any kind of comparison. There were no spring configurations in the list of specimens that would be described by this formula. In both of these published formulae there is a lack of clarity about the interpretation of the effect of the "fixed" arm of the spring.

The derivation of a new emperical formula is beyond the scope of this investigation but a survey of the methods for analyzing structural beams and industrial springs has indicated that a more appropriate formula could be devised.

A. Summary VI:

A specially designed testing machine was used to measure the force-deflection characteristics of a series of spring wire forms that are used in constructing orthodontic appliances. The first group of spring specimens formed a factorial experiment to disclose the effects of length of the legs of a helical loop, number of turns in the loop, and diameter of the loop. The second group of springs was to compare the spring rate of expansion or contraction loop designs. The third group of springs was tested to see how spring rates were effected by the construction and degree-of-restraint of the arms of the helical loop springs.

A more accurate method of reducing the force-deflection measurements to spring rates has been described and used in the whole data-reduction process. A suitable method of analyzing the data has been applied and the important phenomena of interactions between the sources of variation have been revealed. Measures of experimental error have been derived to permit making tests for significance and to tell the reader the degree of reliability of the results.

B. Conclusions:

(1) There are significant interactions between the dimensions of helical loop springs.

(2) The spring rate increases as the legs of a helical loop are shortened. It increases as the diameter of the loop is reduced and as the number of turns is reduced. The spring rate decreases as the amount of free wire in the arms is increased.

(3) Generalizations about the effects of varying the dimensions of the parts of helical loops are of little value when more than one dimension is to be changed.

(4) The only meaningful way to incorporate the various dimensions and characteristics of helical loop springs into a satisfactory expression of their relationship is in a suitable formula. The best one that was observed in the literature was modified to be as follows:

$$\frac{P}{S} = \frac{1}{\sqrt{2}}\left\{\frac{E\,I}{\pi r n L^2 + L r^2 + 1/3 L^3}\right\}$$

L = the length of each arm
r = the radius of the coil
n = the number of turns
E = the modulus of elasticity of the material
I = moment of inertia

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APPROVAL SHEET

The thesis submitted by Dr. Velton C. White has been read and approved by members of the Departments of Anatomy and Oral Biology.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Science.

5-15-63
DATE

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Signature of Adviser